Predicting ocean rogue waves from point measurements: an experimental study

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Abstract

Rogue waves are strong localizations of the wave field that can develop in different branches of physics and engineering such as water or electromagnetic waves. Here, we experimentally quantify the prediction potentials of a comprehensive rogue-wave reduced-order precursors tool that has been recently developed. The laboratory tests have been conducted in two different water wave facilities; in both cases we show that the spontaneous emergence of extreme events is well-predicted through the reported scheme. Due to the interdisciplinary character of the approach, similar studies may be motivated in other nonlinear dispersive media, such nonlinear optics, plasma and solids governed by similar equations allowing the early stage of extreme wave detection.

1 Introduction

Rogue waves, also known as freak waves, are abnormally large waves with crest-to-trough height exceeding eight times the standard deviation of the surrounding surface elevation \cite{1, 2, 3, 4, 5}. Although rare, these waves can have dramatic effects on ships and other ocean structures \cite{6, 7}. Therefore, predicting such extreme events is an important challenging topic in the field of ocean engineering, as well as other fields of wave physics including plasma \cite{8}, solids \cite{9} and optics \cite{10, 11, 12}. In addition, from a mathematical viewpoint the short-term prediction problem of extreme events in nonlinear waves presents particular interest due to the stochastic character of water waves but also the inherent complexity of the governing equations.

Before discussing in details the emphasized prediction tool for rogue waves, we find relevant to make a general statement on the predictability of surface gravity waves: in \cite{13} it has been shown numerically that ocean waves are described by a chaotic system; this implies that after some time (space), the system loses memory of the initial condition and any attempt to perform a deterministic forecast will generally fail. Annenkov and Shrira, \cite{13}, found that such time scale of predictability for typical steepnesses of the ocean waves is of the order of 1000 wave periods. For larger times,

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predictions including rogue wave forecast, can be made only on a statistical bases, i.e., given a wave spectrum and its evolution, the goal is to establish the probability distribution of wave height or wave crest for the given sea-state. This allows one to calculate the probability of encountering a wave whose height is larger than a certain threshold (usually two times the significant wave height), see for example [14]. On shorter time scales, a deterministic prediction of rogue waves is in general possible. In [15] a predictability time scale for rogue waves was estimated through extensive numerical simulations using a phase-resolved high-order spectral technique [16, 17]. It was demonstrated that a time scale for reliable prediction can be $O(10T_p)$ where $T_p$ is the peak period of the spectrum.

For long-crested water waves, statistics are far from Gaussian with heavy tails [18, 19, 20, 21]. In this case, the dominant mechanism for the formation of large waves is finite-time instabilities rising in the form of nonlinear wave focusing [22, 23, 24]. For deep water waves, a manifestation of this focusing is the well-known modulation instability of a plane wave to small sideband perturbations [25, 26]. This instability, which has been demonstrated experimentally already in 1960s [27, 28] and its limiting case more recently [29, 30], generates significantly focused coherent structures by soaking up energy from the nearby field [31, 32, 33]. In this context it is possible and more advantageous to study the dynamics of wave groups (in contrast to individual) waves through reduced-order representations [34, 35, 24], alleviating the direct numerical treatment of the full equations. Depending on the typical dimensions (length, width, height) of the wave group, we may have subsequent nonlinear focusing, which leads to further significant magnification of the wave group height.

The mechanism that leads to the formation of such critical wave groups is the dispersive propagation of waves that results in a continuous mixing of phases between different harmonics (Fig. 1). During this phase, which is the most typical in nonlinear waves (i.e. it occurs most of the time) the dynamics are weakly nonlinear and the conditional statistics are nearly Gaussian [36]. However, if this mixing of waves leads to the formation of a critical wave group, we have nonlinear evolution that leads to rogue wave formation. Such nonlinear evolution can be foreseen using simple precursors that quantify the conditions for nonlinear focusing of the wave group (Fig. 1).

A reduced-order precursor for the prediction of rogue waves has been proposed for uni-directional [36, 37] as well as directional [38] surface gravity waves. The idea behind it comes from combining spectral information for the sea state and information involving the evolution of isolated wave groups to rogue waves. The derived precursors have the form of characteristic patterns that precede rogue waves $O(10T_0)$ ahead, $T_0$ being the dominant wave period. Using field information for the region of interest, the predictive scheme quickly identifies locations where these patterns are present and provides the estimated magnitude of a rogue wave that will occur in the near future, close to this location. The developed scheme is particularly robust given that it relies on the detection of large scale features (having the size of the wave group) utilizing either temporal or spatial measurements. For this reason the scheme does not depend on small scale measurement errors. In addition, it is extremely fast due to the fact that there is no need to calculate any solution of any evolution equation involved in the prediction process.

In this work, we introduce and validate a data-driven prediction scheme based on point measurements, in contrast to field measurements. Similarly to [36, 37], the developed algorithm is formulated for the case of unidirectional rogue waves caused by nonlinear focusing, to capture the deterministic/predictable phase of the evolution associated with nonlinear dynamics. Our starting point is the modified nonlinear Schrödinger Equation (MNLS) [39] formulated as an evolution equation in space rather than in time [40]. The analysis of this universal equation, that can be also
Figure 1: In the typical regime the dominant mechanism is the weakly nonlinear dispersive mixing of phases. After the random formation of a critical wave group the strongly nonlinear dynamics associated with nonlinear focusing can be foreseen through simple precursors.

applied to a wide range of physical media (for instance in optics [41]), allows for the characterization of wave groups or pulses as critical to become rogue or not through single point measurements of the time-series of the surface elevation. We demonstrate the effectiveness of the developed scheme through experimental hydrodynamic data, in the form of time-series. Using multiple realizations of rogue waves, we statistically quantify the accuracy of the developed scheme.

2 Precursors based on point measurements

Our goal is to predict extreme waves in unidirectional wave fields on the surface of deep water, using time measurements at a single point with satisfactory high sampling frequency. The developed scheme consists of an offline, as well as an online, real-time component. For the offline component, we quantify the critical wave groups that evolve to rogue waves using direct numerical solutions of the MNLS equation. Here we employed the MNLS equation for demonstration purposes; the fully nonlinear water wave equations could also be used but the offline component would be computationally more expensive. In the online, real-time component, we identify the coherent wave groups in measurements of a physical, irregular wave time series. We then use the results from the offline component to predict how the measured groups will evolve.

The scheme we discuss here closely follows the ideas presented in [42]. In this case the prediction analysis was based on the availability of field measurements. The algorithm reported in this work predicts future extreme waves from time series measurements of the wave field at a single point. Such formulation yields a tremendous practical payoff, since it allows for the application of the algorithm to experimental data as well as it potential application to more realistic oceanic setups.
2.1 Evolution of isolated, localized groups

We begin by performing an analysis of localized wave groups using the space-time version of the MNLS [39]:

\[
\frac{\partial u}{\partial x} + \frac{2k}{\omega} \frac{\partial u}{\partial t} + i\frac{k}{\omega^2} \frac{\partial^2 u}{\partial t^2} + i\frac{k^3}{\omega} |u|^2 u - \frac{k^3}{\omega} \left( 6|u|^2 \frac{\partial u}{\partial t} + 2u \frac{\partial |u|^2}{\partial t} - 2iu\mathcal{H} \left( \frac{\partial |u|^2}{\partial t} \right) \right) = 0,
\]

where \( u \) is the envelope of the wave train, \( \omega \) is the dominant temporal frequency, related to the wave number \( k \) through the dispersion relation \( \omega^2/g = k \), and \( \mathcal{H} \) is the Hilbert transform, defined in Fourier space as:

\[
\mathcal{F}[\mathcal{H}[f]](\omega) = i \text{sign}(\omega) \mathcal{F}[f](\omega).
\]

The above MNLS equation was derived from the fully nonlinear equations for potential flow on the surface of a deep fluid [40]. The wave field is assumed to be narrow-banded and the steepness small. To leading order, the surface elevation \( \eta(x,t) \) is given by

\[
\eta(x,t) = \Re \left[ u(x,t) \exp \left( i(kx - \omega t) \right) \right];
\]

higher order corrections may also be included, see for instance [43].

While the standard form of MNLS (time-space) can be used to understand how spatially defined wave groups will evolve in future times [36, 37], the above formulation allows us to predict how temporally defined wave groups (over a single point) will evolve in space. For this reason it is an appropriate advantageous formulation in the case where we aim to rely just on one point measurement (over time) in order to predict the occurrence of a rogue wave downstream of the wave propagation. We emphasize that the proposed time-domain analysis and prediction can be also applied to electromagnetic waves [44].

To investigate the evolution of localized wave groups, we consider boundary data of the form

\[
u(x = 0, t) = A_0 \text{sech}(t/\tau_0).
\]

The choice of such function is not related to any special solution of the NLS equation, but rather by the fact that it has the shape of a wave group (a Gaussian shaped function would imply the same type of dynamics). Therefore, we numerically evolve such groups for different amplitudes \( A_0 \) and periods \( \tau_0 \). In fact, for each \((A_0, \tau_0)\) pair, in the case of group focusing, we record the value of the amplitude of the group at maximum focus [45]. In Figure 2, we display the group amplification factor as a function of \( A_0 \) and \( \tau_0 \). Similar to [24], we can notice that indeed some groups focus and increase in amplitude, while others defocus and do not grow. These focusing groups may act as a trigger for the occurrence of extreme waves in unidirectional wave fields, and therefore, we may be able to predict extreme waves in advance by detecting such packets.

2.2 Prediction Methodology

In the proposed prediction scheme the validation will be based on time series data describing the evolution of waves in experimental water wave facilities. This data provides several measurements at different stages of waves evolution for the surface elevation \( \eta \) at different single spatial points. To make a future forecast at probe location \( x^* \) at time \( t^* \) we follow the steps as described below:
Figure 2: Amplification factor for group evolution. An amplification factor of 1 indicates that the group defocuses and does not increase in amplitude. This figure was generated by evolution simulations of the nondimensionalized MNLS.

1. Compute the envelope by Hilbert transform and apply a band pass filter in order to remove the higher harmonics, as suggested in [46, 47], using measurements of $\eta(x^*, t), \, t \leq t^*$.

2. Apply a scale selection algorithm, described in Appendix A, to detect coherent wave groups and their amplitude $A$ and wave group period $\tau_0$.

3. For each group, we estimate the future elevation of the wave field by interpolating the results from the localized wave group numerical experiment (Figure 2).

Note that the above procedure can accurately predict the degree of subsequent magnification of the wave group due to nonlinear focusing. However, apart of a rough estimate on the time required for the nonlinear focusing to occur, it does not provide us with the exact location of the rogue wave focusing.

3 Analysis of two sets of experimental data

Hereafter, we will apply the scheme to two types of experiments performed in different water wave facilities. In the first experimental campaign, the idea is to embed a particular solution of the NLS equation that is known to focus, in an irregular and realistic sea state. For this purpose, we apply to the wave maker a NLS Peregrine-type solution, known to describe nonlinear rogue wave dynamics. However, in this case the boundary conditions launched into the wave maker have been modeled to be embedded into a typical ocean JONSWAP spectrum. More details on the construction methodology can be found in [48]. In this study, the goal is to address the problem if it would be
possible to detect rogue wave solutions at early stage of wave focusing, once embedded in a random sea state.

The second experimental study consists in generating a JONSWAP spectrum with random phases and observes the spontaneous modulation of extreme oceanic waves. Here, the reported scheme is applied to the time series closest to the wave maker in order to establish an early stage of extreme wave event forecast, avoiding any computational effort in simulating their evolution, predicting the rogue wave formation in the water wave facility.

3.1 Critical wave groups embedded in irregular sea configurations

We recall that breathers are exact solutions of the nonlinear Schrödinger equation [3, 46]. Some of them describe the nonlinear stage of classical modulation instability process, namely of a periodically perturbed wave field [49, 50]. The case of infinite modulation period is known as the Peregrine breather [51] that has been so far observed in three different physical systems: optics, hydrodynamics and plasma [52, 29, 8]. The relevance of the Peregrine solution in the rogue wave context is related to its significant amplitude amplification of three and to its double localization in both, time and space.

3.1.1 Description of experiments

The experimental stability analysis of the Peregrine solution is a substantial scientific issue to tackle, if connecting this basic simplified model to be relevant to ocean engineering applications. To achieve this, initial conditions for a hydrodynamic experiment have been constructed, embedding a Peregrine solution into JONSWAP sea states. We recall that a uni-directional JONSWAP sea is defined, satisfying the following spectral distribution [53]:

\[
S(f) = \frac{\alpha}{f^5} \exp \left[ -5 \left( \frac{f_p}{f} \right)^4 \right] \exp \left[ - \frac{(f - f_p)^2}{2\sigma^2 f_p^2} \right]; \tag{4}
\]

the surface displacement can be obtained from the spectrum by:

\[
\eta_{\text{JONSWAP}}(0, t) = \sum_{i=1}^{N} \sqrt{2S(f_n) \Delta f_n} \cos (2\pi f_n t - \phi_n), \tag{5}
\]

with random phases \( \phi_n \in [0, 2\pi) \), [54]. Details of the Fourier space construction methodology are described in detail in [48]. In fact, the wave elevation at \( x = 0 \) (the location of the wave maker) has been constructed to satisfy a JONSWAP sea state configuration with a significant wave height of \( H_s = 0.03 \) and \( H_s = 0.025 \) m as well as a spectral peakedness parameter of \( \gamma = 6 \) and \( \gamma = 3 \), respectively. This allows us to track the evolution of an unstable packet in time and space in irregular conditions while evolving for instance in a water wave facility, rather than assuming spontaneous emergence, as will be discussed in the next section. The experiments have been conducted in a water wave facility with flap-type wave maker. Its length is of 15 m with a width of 1.5 m while the water depth is 1 m, see Figure 3 and [55] for a schematic representation and description.
3.1.2 Assessment of the scheme

In the following, we apply the prediction scheme to the wave tank measurements, related to the experiments of embedded Peregrine models in uni-directional sea state conditions. The results of the first experiment are for $\gamma = 6$, while the second realization has been done for $\gamma = 3$, as described above. The wave propagation as well as the prediction scheme, applied to both experiments, are shown in Fig. 4. Wave groups with predicted wave amplitude that exceeds the rogue wave threshold are noted with red color. Orange, yellow and green colors indicate wave groups with predicted amplitudes that have descending order and below the rogue wave threshold.

First, we can clearly notice the focusing of the initially small in amplitude Peregrine wave packets to extreme waves. On the left hand side of Fig. 4, the case of $\gamma = 6$, the maximal wave height indeed exceeds twice the significant wave height, satisfying the formal definition of ocean rogue waves, whereas in the case depicted on the right-hand side the maximal wave measured was slightly below the latter threshold criteria. Here, we emphasize that the oceanographic definition of rogue waves is based on an ad-hoc approach [3]. Indeed, freak waves having heights that correspond to six times the standard variation could be as dangerous as well. Furthermore, extreme waves satisfying this criterium do not represent a significant threat in the case of low value for the significant wave height.

Note that due to discrete positioning of the wave gauges along the flume, it may be possible that higher amplitude waves have not been captured in the spacing between two wave gauges. Nevertheless, the prediction scheme was clearly successful in detecting the embedded pulsating Peregrine wave packet, see each of the red time windows in Fig. 4, proving the applicability of the method to detect modulationaly unstable wave groups in uni-directional seas. Note that the water wave dynamics in the wave flume is much more complex than described by the NLS and MNLS. In fact, breaking and higher-order nonlinear interactions are inevitable features. The success of the scheme in identifying the unstable wave packets at early stage of focusing proves, however, that the main dynamics can be indeed described by means of weakly nonlinear evolution equations.
3.2 Spontaneous emergence of rogue waves from a JONSWAP spectrum

A time series built from a JONSWAP spectrum is characterized by many wave packets whose amplitudes and widths depend on the total power of the spectrum and on its width, respectively. It has been established that if the spectrum is narrow, the wave packets will have larger correlation lengths and, if they are sufficiently large in amplitude, they can go through a modulation instability process [56], which eventually culminates in a rogue wave. Similarly, with the previous section, the goal here is to establish a priori which of the initial packets will eventually go thorough this process.

3.2.1 Description of the experiments

The data we use here have been collected during an experimental campaign performed at Marintek in Trondheim (Norway) in one of the longest existing water wave flumes. The results of the experiments are collected in the following papers [57, 58, 59]. Here, we report only the main features of the experimental set-up: the length of the flume is 270 m and its width is 10.5 m. The depth of the tank is 10 meters for the first 85 meters, then 5 meters for the rest of the flume. We have employed waves of 1.5 seconds of peak period; this implies that with some good approximations waves can be considered as propagating in infinite water depth, regardless of the mentioned bathymetry variation. A flap-type wave-maker and a sloping beach are located at the beginning and at the far end of the tank so that wave reflection is minimized. The wave surface elevation was measured simultaneously by 19 probes placed at different locations along the flume; conductance wave gauges were used.

3.2.2 Assessment of the scheme for different parameters

In Fig. 5 we present two cases of successful prediction. The moment we have measured through the first probe the elevation of the wavegroup, we are able to predict how the height of the wave group
will evolve and whether it will exceed the rogue wave threshold. The prediction is confirmed by measurements through a probe that is placed further in the wave tank. In Table 1 we summarize the statistics for the prediction scheme. We observe that in all cases of $\gamma$ the prediction is accurate while we miss very few rogue waves. The prediction time, i.e. the duration from when we first predict a particular rogue wave to the time when it is first detected, has $O(10T_0)$ length. This is consistent with the numerical studies in [15, 37].

![Image of predicted wavegroups and rogue waves]

**Figure 5:** Successful prediction of a rogue wave occurring in an irregular wavefield characterized by a JONSWAP spectrum with $\gamma = 3$ (left) and $\gamma = 6$ (right).

**Table 1:** Prediction statistics for rogue waves occurring in a JONSWAP spectrum with different parameters.

<table>
<thead>
<tr>
<th>Parameter $\gamma$</th>
<th>Correct (%)</th>
<th>False Negative (%)</th>
<th>False Positive (%)</th>
<th>Prediction time (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma = 1$</td>
<td>80% (17/19)</td>
<td>10% (2/19)</td>
<td>34% (9)</td>
<td>22.3sec</td>
</tr>
<tr>
<td>$\gamma = 3$</td>
<td>100% (42/42)</td>
<td>0% (0/42)</td>
<td>40% (28)</td>
<td>26.0sec</td>
</tr>
<tr>
<td>$\gamma = 6$</td>
<td>95% (58/61)</td>
<td>5% (3/61)</td>
<td>34% (30)</td>
<td>23.0sec</td>
</tr>
<tr>
<td>All cases</td>
<td>96% (117/122)</td>
<td>5% (5/122)</td>
<td>36% (67)</td>
<td>24.0sec</td>
</tr>
</tbody>
</table>

Despite the good behavior of the algorithm in terms of not missing extreme events, it has a relatively large false-positive rate. We attribute this characteristic to the existence of noise or other imperfections of wave profiles, that are for instance a result of wave breaking, which are inevitable in this experimental setup and thus, may lead to overestimation of the height of the wavegroup. Moreover, it is possible that the actual false positive rate is lower than 36%, since we only have measurements of the wave field at the location of the probes while a wave group may only exceed the extreme height threshold at a location where we have not been monitoring along the wave flume. This would be then subsequently classified as a false positive.
4 Conclusions

To summarize, we have applied a reduced-order predictive scheme for extreme events, based on the dynamics of NLS, to two types of laboratory data: in the first the extreme events have been modeled to arise from seeded unstable deterministic breather dynamics, embedded in a JONSWAP sea state, while in the second the extreme events have emerged spontaneously from the JONSWAP wave field. Considering the fact that during the laboratory experiments the wave profiles have been measured discretely along the flume, some of the false positive predictions may be still regarded as successful. Overall, the prediction scheme could reach a very high success rate and thus, proves its efficiency and its broad range of applications, particularly, in the field of ocean engineering as well as nonlinear fibre optics, both media in which the dynamics of extreme waves is of significant relevance for the sake of prevention and applications. Nevertheless, further studies are required to assess applicability for instance to directional seas [21]. Indeed, the uni-directional wave propagation can be related only to swell propagation, whereas, sea dynamics can be more complex in nature. Spatial measuring techniques using stereo camera are promising in capturing water surface distributions [60]. On the other hand, applications to other nonlinear dispersive media are inevitable. Knowing that the uni-directional wave propagation in Kerr media follows NLS-type evolution equations with better accuracy as for the case for water waves, (as the degree of nonlinearity of electromagnetic waves propagating in nonlinear fiber optics can be accurately controlled by the Kerr medium [61, 62], while breaking thresholds are much higher [63] compared to water waves) , a better accuracy of the scheme is expected.

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References


