Reduced order modeling of hydrodynamic interactions between a submarine and unmanned underwater vehicle using non-myopic multi-fidelity active learning

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Abstract

Several efforts have been dedicated to developing computational tools capable of predicting the hydrodynamic forces and moments of Unmanned Underwater Vehicles (UUVs). However, there is no method at the moment that allows for real-time computational modeling of all the complex hydrodynamic interaction forces and moments that a UUV experiences when operating in close proximity to a moving submarine. Real-time modeling of these hydrodynamic interactions is essential to simulate the motion required to launch and recover UUVs from submarines. Potential flow models are often fast enough to be used in real time, but lack the accuracy of Computational Fluid Dynamics (CFD) simulations, which often take hours or days to solve. Here, we formulate the problem in the context of machine learning, specifically active learning. The goal is to develop a surrogate model capable of predicting the UUV and submarine hydrodynamic interactions in real time using a very small number of carefully selected CFD simulations. We introduce a new active learning framework called Non-Myopic Multi-Fidelity Active Learning for Gaussian Process (GP) regression that accelerates the convergence of the surrogate model by utilizing the low cost of the low fidelity simulations to explore the domain, as well as optimally selected high fidelity simulations to improve the model accuracy. The resulting surrogate model can be integrated into UUV control and autonomy systems and motion simulators to further enable UUV launch and recovery from submarines. This new active learning method may also be used to create higher accuracy and lower cost surrogate models in other real world applications.

Keywords— Reduced order modeling of hydrodynamic interactions; Unmanned underwater vehicles; Non-myopic active learning; Multi-fidelity Gaussian process regression.

1 Introduction

In order to enable the launch and recovery of UUVs from submarines, a UUV needs to be able to overcome the hydrodynamic interaction forces and moments between the two vehicles [1]. These hydrodynamic interactions are often predicted using high fidelity CFD modeling due to its high accuracy. While most CFD simulations take hours to days to complete, a UUV control system needs to respond within milliseconds in order to maintain the desired trajectory. As such, the UUV needs to determine the hydrodynamic interaction forces and moments in real time based on its position, heading, speed, and proximity to its desired path.

Modeling loads on the UUV can be performed with a variety of computational models that resolve the governing fluid equations. These range from low fidelity and low cost models to more expensive and more accurate models. Specifically, low fidelity models, such as potential flow solvers use simplified physics which neglects viscosity, skin friction, boundary layer development, flow separation, and leads to the d’Alembert paradox which predicts zero drag on a UUV moving at constant velocity [2, 3]. These low fidelity solvers are often improved and supplemented with simple parametric models to help overcome these weaknesses and often have the capability to be solved in real time [4]. However, they
ultimately lack the accuracy needed to model the complex real world hydrodynamic interactions to enable UUV launch and recovery operations. This can be achieved by CFD solvers that rigorously model all the important fluid mechanics phenomena, but they have significant computational cost. Because the required accuracy of the CFD is not capable of being delivered in real time, a surrogate model is needed that is capable of being implemented in real time and has the accuracy of the CFD.

A typical approach for building a surrogate model is to collect high fidelity data, i.e. from expensive and highly accurate CFD solvers and apply reduced order modeling ideas. However, even in an offline setting (i.e. just to produce training data), the computational cost of CFD can be prohibitive, given that the parameter space for a UUV is pretty significant. For such a case, an alternative is to combine a few, carefully selected simulations from a CFD model with plenty of low fidelity computations, e.g. from a potential flow solver. Integrating various fidelity models into one surrogate is known as multi-fidelity modeling [5, 6]. By leveraging data from a lower cost and less accurate model with data from a high fidelity model, the accuracy of the surrogate model can be improved without the need for an excessive number of high fidelity simulations. This results in a significant reduction of the computational cost of the surrogate model without sacrificing accuracy.

An important question for developing accurate surrogate models is the selection of the most informative training data, i.e. what CFD simulations one should perform to get the most important information. This can be achieved by employing active learning, a type of machine learning data sampling method in which the algorithm is able to determine the optimal set of input parameters for which the next simulation should be performed [7, 8, 9]. Typical active learning algorithms are characterized by myopia, or nearsightedness: a condition in which lack of foresight can inhibit the ability of a sampling algorithm to select the optimal sampling location for the surrogate model [10, 11]. For example, a game exists where an unknown random number is selected between 0 and 100. The object of the game is to minimize the value between a number chosen by the participant and the unknown random number. In this instance, the optimal number to select would be 50, halfway between the two endpoints. This ensures that the maximum error between any random number and the selected value is 50. Now assume the participant is allowed to select a second number. Once 50 is selected, the next optimal value would be either 25 or 75. However, neither of these two options reduces the maximum error because there is still the potential to be off by 50. This problem exists because of myopia. Originally, the participant is only considering selecting one single number. If the selection method is non-myopic, there are different optimal values chosen. By knowing beforehand that two selections would be made, the optimal selections would be at 25 and 75. By selecting these two points, the maximum value between any random number and one of the selected values would be 25. This illustrates how having the foresight of knowing about future sampling characteristics provides a different set of optimal sampling locations.

This paper introduces the use of a non-myopic multi-fidelity active learning GP regression algorithm for reduced order modeling and compares it with standard myopic active learning techniques. We begin with a quick review of Gaussian process regression, multi-fidelity modeling, and active learning in a myopic and non-myopic setup. In section 3, we present the formulation and implementation of the method, and subsequently, in section 4 and 5, we examine its relative advantages in the context of standard benchmark functions and prototype problems for scalar and vector outputs. In section 6, we examine its performance and demonstrate its favorable properties in the context of reduced order modeling for UUV and submarine hydrodynamic interactions.

2 Background

2.1 Gaussian Process Regression

The purpose of GP regression is to develop a surrogate or reduced order model that is capable of predicting the value of dependent variables based on the input of independent variables. However, in contrast to typical regression methods, GP provides rigorous estimates for the epistemic uncertainty of the derived model, i.e. errors due to lack of data. The GP regression model can be expressed as a random function,
\[ y = f(x) + \epsilon \]  \hspace{1cm} (1)

where \( x \in \mathbb{R}^d \), \( \epsilon \sim \mathcal{N}(0, \sigma^2) \) represents the noise of the model and the random function \( f \) follows a Gaussian distribution with prescribed mean and covariance function \([12]:\)

\[ f(x) \sim \mathcal{GP}(\mu(x), k(x, x')) \] \hspace{1cm} (2)

where \( \mu(x) \) is the mean and \( k(x, x') \) the covariance:

\[ \mu(x) = \mathbb{E}[f(x)] \] \hspace{1cm} (3)

\[ k(x, x') = \mathbb{E}[(f(x) - \mu(x))(f(x') - \mu(x'))] \] \hspace{1cm} (4)

There are many different covariance functions (or kernels) that are often used in GP models. Some of the more popular kernels include the white noise kernel, squared exponential kernel, rational quadratic kernel, and the periodic kernel \([13]\). Rather than explore the impact of these different kernels, this study uses the popular radial basis function (RBF) kernel with automatic relevance determination:

\[ k(x, x') = \exp \left( \frac{-(x - x')^T \lambda^{-1}(x - x')}{2} \right) \] \hspace{1cm} (5)

where \( \lambda \) is the diagonal matrix containing the length scales of each input dimension. This kernel is selected because it simulates a Bayesian linear regression model with an infinite number of basis functions. In other words, this kernel can be formed from a linear combination of an infinite number of Gaussian-shaped basis functions \([12]\). Because an infinite number of these basis functions can determine the form of any (sufficiently smooth) output function, this method is well suited for this study with an unknown form of the different output functions. Automatic relevance determination is used because it enables the GP regression kernel to have different length scales for each input dimension.

GP regression is ultimately used to calculate the predicted mean \( y(X_*) \) and covariance \( K_{yy}(X_*, X'_*) \) when conditioning on a set of input-output data pairs. In particular, the model is trained with a data set \( \mathcal{D} = \{x_i, y_i\}_{i=1}^n \), where \( n \) is the number of samples. We also use the notation \( X = [x_1, ..., x_n] \in \mathbb{R}^{d \times n} \) and \( y = [y_1, ..., y_n] \in \mathbb{R}^n \). Likewise, \( X_* = [x_{s1}, ..., x_{sn}] \in \mathbb{R}^{d \times m} \), is a set of \( m \) locations within the \( d \)-dimension domain for which a prediction is desired. Equations (6) and (7) determine the predicted mean and covariance at a set of points \( X_* \): \([12]:\)

\[ y(X_*) = K(X_*, X)[K(X, X) + \sigma_n^2 I]^{-1}y \] \hspace{1cm} (6)

\[ K_{yy}(X_*, X'_*) = K(X_*, X'_*) - K(X_*, X)[K(X, X) + \sigma_n^2 I]^{-1}K(X, X'_*) \] \hspace{1cm} (7)

The term \( \sigma_n^2 \) represents the aleatoric uncertainty in the training samples. It is a hyperparameter that is optimized using gradient descent methods to improve the predictive capabilities of the GP regression \([12]\). Additionally, it helps ensure the matrix in brackets in equations (6) and (7) is well conditioned.

### 2.2 Active Learning

For many problems, like the one considered here, the cost of obtaining accurate training data for a GP regression is very large. As such, each new data point is selected sequentially and methodically, so that it provides the most improvement to the surrogate model. This type of data sampling method in which the algorithm is able to determine the optimal set of input parameters for the next sample is called active learning or optimal experimental design \([7, 8]\).

Suppose we have a GP regression model \( y_{n-1}(x) \) with an error of \( \sigma_{n-1}(x) = \sqrt{K_{yy}(x, x)} \) trained from a data set with input vectors \( \{x_1, x_2, ..., x_{n-1}\} \). The aim in active learning is to use the predicted mean and error estimate, to optimize the selection of the next sample, \( x^* \). Specifically, active learning uses
what is called an acquisition function \( q(x|y_{n-1}, \sigma_{n-1}) \) in order to determine this next optimal sample \( x^* \) as below:

\[
x^* = \arg \min q(x|y_{n-1}, \sigma_{n-1}).
\]  

(8)

Once this optimal sampling location \( x^* \) is determined, the sample is taken at that point and the new data is added to the existing data set \( D_n = \{D_{n-1} \cup (x^*, y^*)\} \). The new surrogate mean \( y_n(x) \) and error \( \sigma_n(x) \) are determined and the process is iterated as necessary. Figure 1 illustrates the active sampling process.

Figure 1: Demonstration of active sampling method of determining the optimal sample location and updating the predicted mean of the surrogate model.

In figure 1, the optimal sampling location \( x^* \) is determined by locating the point in the domain with the largest model error \( \sigma_{n-1} \). This popular acquisition function is known as Uncertainty Sampling (US) and is shown in equation (9), [9].

\[
q_{US}(x) = \sigma^2(x)
\]  

(9)

The US acquisition function is widely used because it is intuitive, robust, broadly applicable, inexpensive to compute, and has analytical gradients, which allows the use of gradient-based optimizers so it becomes considerably more efficient than other acquisition functions [14]. There are many different acquisition functions like integrated variance reduction, input-weighted integrated variance reduction, mutual information, and likelihood-weighted acquisition functions [9]. However, for the purpose of this study in which non-myopic active learning is explored, only the US acquisition function is used due to its robustness.

### 2.3 Myopic versus Non-Myopic Active Learning Algorithms

Typical active learning sampling methods are myopic, meaning that they only consider a single step into the future when selecting the next optimal sampling location [10]. Once a location is selected, the output is evaluated at that single point and this new data is added to the existing data set. In particular, when a new single location \( x_n \) is determined using active learning criteria, the output \( y_n \) is computed from a simulation or experiment, and the data set is augmented with the new information, i.e. \( D_n = \{D_{n-1} \cup (x_n, y_n)\} \). Figure 2 illustrates this myopic sampling algorithm.
While this myopic approach is often used in practice, there are other non-myopic approaches that provide solutions to the multi-step look-ahead problem with better results than a myopic approach [10, 11]. Specifically, a non-myopic approach enables the algorithm to determine the next optimal sampling location based on the influence of several future potential sampling locations. This allows the algorithm to select the optimal sampling location with the knowledge about how the many future sampling locations may influence the next sample. This influence of future evaluations on current sampling locations is illustrated in figure 3 using a blue line.

There are many different non-myopic algorithms that use various acquisition functions to evaluate the impact of future samples [10, 11, 15, 16]. These acquisition functions vary in how they evaluate and utilize the expected value of the future samples on the model. However, all of these non-myopic sampling methods are only used for single fidelity surrogate models and do not offer a means to which non-myopic methods can be expanded to operate between models of different fidelity. In this study, we explore a new sampling method by bridging the gap between non-myopic sampling and multi-fidelity GP modeling to achieve improvements in lowering the cost and increasing the accuracy of high fidelity simulators.

2.4 Multi-Fidelity Modeling

Multi-fidelity GP modeling uses multiple separate simulators or experiments to develop the surrogate model. Like in the present context, there is a high fidelity model which is computationally expensive to use and a low fidelity model which requires much less computational effort to perform. In the context
of this work, the high fidelity simulation is CFD and the low fidelity simulation is a potential flow solver. Because of the different assumptions and physics being modeled, the CFD and potential flow simulators produce different results for any given sampling location. These results are stored in a high fidelity solver. Because of the different assumptions and physics being modeled, the CFD and potential flow simulators produce different results for any given sampling location. These results are stored in a high fidelity and low fidelity simulation is CFD and the low fidelity simulation is a potential flow solver. These other schemes include a deep GP in which the high fidelity and low fidelity models. This scaling factor is set to one for the purposes of this study because there is no scaling needed between CFD and potential flow simulators. Using a multi-fidelity approach provides the benefit of improving the accuracy and cost by combining expensive accurate high-fidelity data with cheaper and less accurate low-fidelity data [6]. This scheme and the others can be used recursively to account for more than two levels of fidelity, but only two levels are used for this study. There are other schemes to account for multi-fidelity modeling besides the first-order auto-regressive co-kriging scheme. These other schemes include a deep GP in which the scaling factor is replaced with an unknown function $z(f_{LF}(x))$ which maps the difference between the low and high fidelity models [17]. This function $z$ is often another GP regression, which is why this is often called deep GP regression, but the added layer of GP regression comes at a steep computational price. Another scheme is the nonlinear auto-regressive multi-fidelity GP regression scheme in which a higher dimension GP regression model is created that jointly relates the input space and the outputs of the lower fidelity level to the output of the higher fidelity model. Once again, this modification to the multi-fidelity modeling scheme increases the computational cost of the model [18].

As such, the first-order auto-regressive co-kriging scheme is used for this study due to its low cost and ease of implementation [17, 18, 6]. Equation (10) implies the Markov property: given $f_{LF}(X_{\ast})$ we can learn nothing more about $f_{HF}(X_{\ast})$ from any other model output $f_{LF}(X')$ for $X_{\ast} \neq X'$, i.e. $\text{cov}\{f_{HF}(X_{\ast}), f_{LF}(X') \mid f_{LF}(X)\} = 0$, [6].

This allows for the following definition of the high fidelity $f_{HF}(X_{\ast})$, low fidelity $f_{LF}(X_{\ast})$, multi-fidelity $f_{MF}(X_{\ast})$, and $\delta(X_{\ast})$ GP regression models:

$$f_{HF}(X_{\ast}) \sim \mathcal{GP}(y_{HF}(X_{\ast}), K_{yy,HF}(X_{\ast}, X_{\ast}'))$$
$$f_{LF}(X_{\ast}) \sim \mathcal{GP}(y_{LF}(X_{\ast}), K_{yy,LF}(X_{\ast}, X_{\ast}'))$$
$$\delta(X_{\ast}) \sim \mathcal{GP}(y_{s}(X_{\ast}), K_{ys,\delta}(X_{\ast}, X_{\ast}'))$$
$$f_{MF}(X_{\ast}) \sim \mathcal{GP}(y_{MF}(X_{\ast}), K_{yy,MF}(X_{\ast}, X_{\ast}'))$$

where $y_{MF}(X_{\ast}) = y_{LF}(X_{\ast}) + y_{s}(X_{\ast})$

and $K_{yy,MF}(X_{\ast}, X_{\ast}') = K_{yy,HF}(X_{\ast}, X_{\ast}') = K_{yy,\delta}(X_{\ast}, X_{\ast}')$

By expanding equations (2), (6), (7), and (10) to account for the multi-fidelity modeling approach, the predicted mean and covariance of the low and high fidelity GP models are derived. Additionally, the $\delta(X_{\ast})$ GP regression model from equation (10) has a predicted mean and covariance listed in equation (12). In order to determine the mean of the $\delta(X_{\ast})$ GP regression model, a vector of the outputs $y_{s}$ is needed. This is found by using the difference between the high fidelity and low fidelity outputs for each of the $n$ high fidelity samples at the corresponding low fidelity sample locations as shown in equation (15). This requires that the samples of the high fidelity data set are a subset within the low fidelity data set, i.e. $X_{LF} \subseteq X_{HF}$.

$$y_{HF}(X_{\ast}) = K(X_{\ast}, X_{HF})[K(X_{HF}, X_{HF}) + \sigma_n^2 I]^{-1}y_{HF}$$
$$K_{yy, HF}(X_{\ast}, X_{\ast}') = K(X_{\ast}, X_{\ast}') - K(X_{\ast}, X_{HF})[K(X_{HF}, X_{HF}) + \sigma_n^2 I]^{-1}K(X_{HF}, X_{\ast}')$$

$$\sigma_{HF}(X_{\ast}) = \sqrt{K_{yy, HF}(X_{\ast}, X_{\ast})}$$

and

$$y_{LF}(X_{\ast}) = K(X_{\ast}, X_{LF})[K(X_{LF}, X_{LF}) + \sigma_n^2 I]^{-1}y_{LF}$$
$$K_{yy, LF}(X_{\ast}, X_{\ast}') = K(X_{\ast}, X_{\ast}') - K(X_{\ast}, X_{LF})[K(X_{LF}, X_{LF}) + \sigma_n^2 I]^{-1}K(X_{LF}, X_{\ast}')$$

$$\sigma_{LF}(X_{\ast}) = \sqrt{K_{yy, LF}(X_{\ast}, X_{\ast})}$$
and

\[ y_\delta(X_\ast) = K(X_\ast, X_{HF})[K(X_{HF}, X_{HF}) + \sigma^2_\delta I]^{-1}y_\delta \]
\[ K_{yy,\delta}(X_\ast, X_\ast) = K_{yy, HF}(X_\ast, X_\ast) \]
\[ \sigma_\delta(X_\ast) = \sqrt{K_{yy,\delta}(X_\ast, X_\ast)} \]  \hspace{1cm} (14)

and

\[ y_\delta = \{y_{HF,i} - y_{LF,i}\}_{i=1}^n \]  \hspace{1cm} (15)

Equations (11) through (15) are the framework for multi-fidelity GP regression. This framework provides the grounds for which the non-nyopic active sampling concept can be incorporated into a multi-fidelity GP regression scheme to improve the cost and accuracy of the surrogate model.

3 Non-Myopic Multi-Fidelity Active Learning

3.1 Inter-Model Acquisition Function

Motivated by the advantage of producing a low cost and high accuracy surrogate model using the multi-fidelity framework, as well as a non-nyopic setup for active learning, we proceed with the formulation of a non-nyopic multi-fidelity active learning algorithm. Before the algorithm can be formalized, we must introduce a new type of acquisition function called an inter-model acquisition function \( q_{IM} \). It is used to determine the next location for the high fidelity simulation to be performed. This new type of acquisition function is unique because \( q_{IM} \) accounts for the statistics of multiple GP regression models with differing fidelity, rather than a single fidelity GP regression model. The general form of the inter-model acquisition function is as follows:

\[ q_{IM}(X_\ast) = f(y_{HF}(X_\ast), \sigma_{HF}(X_\ast) = \sigma_\delta(X_\ast), y_{LF}(X_\ast), \sigma_{LF}(X_\ast), y_\delta(X_\ast)). \]  \hspace{1cm} (16)

In order to formulate an effective inter-model acquisition function, consider an example high fidelity and a multi-fidelity GP regression model. Recall from equation (11) that \( y_{MF}(X_\ast) = y_{LF}(X_\ast) + y_\delta(X_\ast) \) and \( \sigma_{MF}(X_\ast) = \sigma_{HF}(X_\ast) = \sigma_\delta(X_\ast) \). Figure 4 illustrates a high fidelity and a multi-fidelity GP regression model from equation (11) with three different low fidelity active samples.
(a) Low Fidelity Active Sampling: The high (blue) and multi-fidelity (orange) GP regression models are shown with the next three low fidelity sampling locations.

(b) Inter-Model Active Sampling: The high fidelity sampling location is determined by considering how the multi-fidelity model is impacted by the new low fidelity samples.

Figure 4: Inter-Model Acquisition Function: The location of the next high fidelity sample is determined by considering the difference between the predicted mean of the high and multi-fidelity GP regression models.

In the example in figure 4, the first two low fidelity active samples have little impact on the mean multi-fidelity GP regression model. This is because the multi-fidelity GP regression model is close to the ground truth in these sampling locations so little improvement is made. The third low fidelity sample has a large impact on the multi-fidelity GP regression model because the ground truth is farther away from the prediction. For this reason, the inter-model acquisition function in use for this study calculates the absolute difference between the predicted mean of the high and multi-fidelity GP regression models. This is referred to as the absolute difference inter-model acquisition function and is defined in equation (17). The high fidelity sample is selected at the location in the domain where the maximum absolute difference between the mean of the high and multi-fidelity GP model exists. This is chosen to capitalize on the exploration of the low fidelity sampling. If this difference is the result of a divergence between the low and high fidelity simulations, rather than an optimal location where the high fidelity GP model could be improved, then the $\delta(X_s)$ GP model from equations (10) and (11) is updated to account for this discrepancy. This correction allows the multi-fidelity GP regression model to provide greater opportunities to identify optimal locations for future high fidelity samples.

$$q_{AD}(X_s) = |y_{HF}(X_s) - y_{MF}(X_s)| = |y_{HF}(X_s) - (y_{LF}(X_s) + y_\delta(X_s))|$$

$$x_{HF}^* = \arg \max q_{AD}(X_s) = \arg \max |y_{HF}(X_s) - y_{MF}(X_s)|$$

Because the low fidelity model can be sampled many times before sampling a high fidelity data point, the non-myopic characteristic of the acquisition function emerges. It considers how the low fidelity model changes as it iterates through many future low fidelity samples and uses this information to determine the location of the next high fidelity sample. This non-myopia allows for more robust exploration of the domain in the low fidelity regime before a high fidelity sample is taken. This can
help identify inaccuracies in the high fidelity regime.

3.2 Non-Myopic Multi-Fidelity (NMMF) Active Learning Algorithm

By combining the multi-fidelity GP regression framework with the non-myopic approach for active learning, we proceed with the formulation of a non-myopic multi-fidelity active learning algorithm. It consists of the following steps and the pseudo-code is listed in Algorithm 1:

1. Begin with a small number of high and low fidelity simulations performed as bootstraps, i.e. \( D_{HF} \) and \( D_{LF} \). These bootstraps are performed at the same \( X \) locations within the domain but result in different outputs, i.e. \( y_{HF} \) and \( y_{LF} \). These bootstrapped data sets are used to determine \( D_\delta = \{ X, y_\delta \} \) from equation (15). Next, GP regression is performed on \( D_{HF}, D_{LF}, D_\delta \) to obtain \( y_{LF}, y_{HF}, y_\delta, \sigma_{LF}, \) and \( \sigma_{HF} = \sigma_\delta \) using equations (12), (13), and (14).

2. Perform active learning to select a sample from the low fidelity model using equation (8) and run the corresponding low fidelity simulation. The uncertainty sampling acquisition function in equation (9) is used due to its robustness. After the sample has been selected and simulated, GP regression is performed using equation (13) to find the new \( y_{LF} \) and \( \sigma_{LF} \). This low fidelity sampling is repeated for a set number of iterations. This search provides new information about the low fidelity GP regression model at many new points without having the cost of running multiple high fidelity simulations.

3. Select the high fidelity sample using equation (17) and perform the high fidelity simulation at this sample location. This inter-model acquisition function is non-myopic because it considers how the low fidelity model evolves through multiple samples and uses this information to select the high fidelity sampling location. Next, perform a low fidelity simulation at the location of the high fidelity sample and add it to the low fidelity data set \( D_{LF} \).

4. Remove any low fidelity samples from the low fidelity data set \( D_{LF} \) that are not at locations where high fidelity samples are also taken. This prevents the low fidelity data set from becoming too large to perform the inverse matrix operation in equation (13) and from having a negative impact on the GP regression. Use equation (15) to update \( D_\delta \) with the new high fidelity sample. Finally, perform GP regression on \( D_{HF}, D_{LF}, D_\delta \) to obtain \( y_{LF}, y_{HF}, y_\delta, \sigma_{LF}, \) and \( \sigma_{HF} = \sigma_\delta \) using equations (12), (13), and (14).

5. Repeat steps 2 through 4 until the desired number of high fidelity samples is taken.

The pseudo-code in Algorithm 1 outlines the non-myopic multi-fidelity active learning algorithm for GP regression.
Two different prototype problems are selected in a multi-fidelity setup with a number of dimensions into the low fidelity simulators to improve their accuracy. Additionally, the results of the multi-fidelity GP model could be integrated directly into the low fidelity simulators to improve their accuracy.

While the multi-fidelity GP regression model serves a vital role in the active learning process, there are additional benefits that come from using this approach. Because this multi-fidelity GP model is used to find the relationship between the high fidelity and low fidelity simulators, this difference could be used by simulation developers to better understand the limitations of low fidelity simulators. There may be portions of the input space in which the low fidelity model is suitable for many purposes. The multi-fidelity GP model helps quantify the accuracy of the low fidelity model with respect to the high fidelity model. Additionally, the results of the multi-fidelity GP model could be integrated directly into the low fidelity simulators to improve their accuracy.

3.3 Advantages of Multi-Fidelity Gaussian Process Regression Model

While the multi-fidelity GP regression model serves a vital role in the active learning process, there are additional benefits that come from using this approach. Because this multi-fidelity GP model is used to find the relationship between the high fidelity and low fidelity simulators, this difference could be used by simulation developers to better understand the limitations of low fidelity simulators. There may be portions of the input space in which the low fidelity model is suitable for many purposes. The multi-fidelity GP model helps quantify the accuracy of the low fidelity model with respect to the high fidelity model. Additionally, the results of the multi-fidelity GP model could be integrated directly into the low fidelity simulators to improve their accuracy.

4 Evaluation of NMMF Active Learning Algorithm

To assess the performance of the NMMF active learning algorithm compared to other state of the art algorithms, a large number of high fidelity simulations needs to be performed. For this reason, we first demonstrate its advantages on some test problems and then we apply it to CFD problems. Specifically, the performance of the developed algorithm is compared to that of the traditional myopic multi-fidelity sampling method, as well as the standard sampling method using the US acquisition function for the high fidelity model.

Two different prototype problems are selected in a multi-fidelity setup with a number of dimensions close to that of the UUV and submarine hydrodynamics problem. For both of the problems in this section, the number of low fidelity samples that are performed before selecting the high fidelity sampling location is held constant at 10. The error in use to evaluate the accuracy of the surrogates is the Mean Absolute Percentage Error (MAPE). This is chosen because it normalizes the error, rather than looking at absolute error alone. The MAPE is defined in the following equation in terms of the predicted quantity, $y_p$, and its exact value $y_e$: 

$$MAPE = \frac{1}{n} \sum_{i=1}^{n} \left| \frac{y_{p,i} - y_{e,i}}{y_{e,i}} \right|$$ (18)
4.1 3 DOF trebuchet with a hinged counterweight and sling

We first consider the simulation of projectile range from a three degree of freedom trebuchet with a hinged counterweight and sling [19, 20, 21]. Figure 5 illustrates the mechanical system of the simulation.

Using Lagrangian mechanics, the equations of motion outlined in equation (19) are derived to simulate the motion of the trebuchet using the variables defined in figure 5. These equations of motion are coupled and highly non-linear. These are the types of problems for which surrogate models are often created.

\[
\begin{bmatrix}
  m_1 l_1^2 + m_2 l_2^2 + m_{arm} l_{arm}^2 + I_{arm} & m_1 l_1 l_4 \cos(\theta - \phi) & -m_2 l_2 l_3 \cos(\theta - \phi) \\
  m_1 l_1 l_4 \cos(\theta - \phi) & m_1 l_4^2 & 0 \\
  -m_2 l_2 l_3 \cos(\theta - \phi) & 0 & m_2 l_3^2 \\
\end{bmatrix} \begin{bmatrix}
  \ddot{\theta} \\
  \ddot{\phi} \\
  \ddot{\psi}
\end{bmatrix} =
\begin{bmatrix}
  -m_1 l_1 \dot{\theta}^2 \sin(\theta - \phi) + m_2 l_2 \dot{\psi}^2 \sin(\theta - \phi) - (m_1 l_1 - m_2 l_2 - m_{arm} l_{arm}) g \cos(\theta) \\
  m_1 l_1 l_4 \dot{\theta}^2 \sin(\theta - \phi) - m_1 l_4 g \cos(\phi) \\
  m_2 l_2 l_3 \dot{\psi}^2 \sin(\theta - \phi) - m_2 l_3 g \cos(\psi)
\end{bmatrix}
\]

\hspace{1cm} (19)

where \( m_{arm} = f(l_1, l_2, constants) \), \( l_{arm} = f(m_{arm}, l_1, l_2, constants) \), and \( I_{arm} = f(m_{arm}, l_1, l_2, constants) \).

The initial conditions in use to solve the system of equations are listed below:

\[
\begin{align*}
  \theta_0 &= \sin^{-1}\left(\frac{h_0}{l_2}\right) , \\
  \phi_0 &= -\frac{\pi}{2} , \\
  \psi_0 &= 0 , \\
  \dot{\theta}_0 &= 0 , \\
  \dot{\phi}_0 &= 0 , \\
  \dot{\psi}_0 &= 0
\end{align*}
\]

The projectile is released at a constant launch hook angle \( \beta \). After the projectile is released, the range of the projectile is determined using standard Newtonian projectile motion physics which neglects air resistance.

These equations of motion have the potential to allow for a higher dimension domain by allowing more parameters to vary, like the launch hook angle \( \beta \) or other trebuchet arm moment of inertia parameters like material density or thickness of the arm. Likewise, the domain could be reduced by fixing certain input variables to a constant value. The domain for this problem is selected to be six-dimensional and the range of the various input variables is as follows:
\[ l_1 \in [0.5, 1.5] \]
\[ l_2 \in [3.5, 4.5] \]
\[ l_3 \in [3.0, 4.0] \]
\[ l_4 \in [0.1, 1.0] \]
\[ h_0 \in [3.0, 3.5] \]
\[ m_1 \in [40, 400] \]

A simpler two degree of freedom simulation is used for the low fidelity model of the trebuchet. For this model, the counterweight is not hinged. Instead, it is mounted directly on the end of the trebuchet rotating arm. This simplifies the equations of motion by removing the influence of the \( l_4 \) and \( \phi \) variables.

Figure 6 shows the performance of the different methods on the three degree of freedom trebuchet problem. The entire process is repeated 100 times to reduce the variance between experiments.

![Figure 6: Comparison of non-myopic multi-fidelity active learning algorithm against other active sampling algorithms on the three degree of freedom trebuchet.](image)

We first note that the addition of the extra low-fidelity data points significantly improves the surrogate model. In addition, the non-myopic multi-fidelity sampling method produces a more accurate model with fewer high-fidelity experiments than the other sampling methods, although the benefits in this case are not substantial.

### 4.2 Borehole Function

The borehole function is an eight-dimensional highly non-linear equation developed by Harper and Gupta [22]. This equation is used to determine the volumetric flow rate through a borehole that is drilled through an upper aquifer, a nuclear waste repository, and into a lower aquifer. This function has been used in literature to evaluate the performance of computer models. Xiong also developed a low fidelity approximation of this model which enables the borehole function to also evaluate multi-fidelity models [23]. The multi-fidelity borehole function, listed in equations (22) and (23), is used to evaluate the performance of the non-myopic multi-fidelity sampling algorithm.

\[
y_{HF}(x) = \frac{2\pi T_u (H_u - H_l)}{\ln (r/r_w)} \left( 1 + \frac{2LT_u}{\ln (r/r_w)r_w^2K_w} + \frac{T_u}{T_l} \right)^{-1}
\]

\[
y_{LF}(x) = \frac{5T_u (H_u - H_l)}{\ln (r/r_w)} \left( 1.5 + \frac{2LT_u}{\ln (r/r_w)r_w^2K_w} + \frac{T_u}{T_l} \right)^{-1}
\]
Equation (24) lists the domain of the various input variables.

\[
\begin{align*}
    r_w & \in [0.05, 0.15] \\
    r & \in [100, 50000] \\
    T_u & \in [63070, 115600] \\
    H_u & \in [990, 1110] \\
    T_l & \in [63.1, 116] \\
    H_l & \in [700, 820] \\
    L & \in [1120, 1680] \\
    K_w & \in [9855, 12045]
\end{align*}
\] (24)

The considered GP sampling methods are evaluated on the borehole function 450 times to reduce the variance of the results. Figure 7 shows how each method performs on the borehole function test case.

Figure 7: Comparison of non-myopic multi-fidelity active learning algorithm against other active sampling algorithms on the borehole function.

The NMMF sampling method is able to produce the lowest model error in the majority of the solution space. Initially, this method has the highest error, but it rapidly improves to the most accurate sampling scheme, approximately after 30 to 40 simulations are conducted. This is likely because the low and high fidelity models have a larger difference than in the other test cases. This means that the initial high fidelity samples are likely used to quantify the difference \( \delta(X_*) \) (equations (10) and (11)) between the high and low fidelity models, rather than better explore the domain. Once this difference is well explored and quantified, the exploratory benefits of the NMMF algorithm quickly outperform the other two sampling methods.

5 Adaptations for multi-dimensional outputs

The considered prototype problems demonstrate the effectiveness of the NMMF active learning algorithm in setups where the output is a scalar quantity. However, for the submarine and UUV hydrodynamic interaction problem, the hydrodynamic interaction surrogate model has vector outputs. One can address this issue by simply building three separate surrogate models, one for each output. However, having multiple outputs creates a new obstacle. When a high fidelity CFD simulation is run at a single location in the domain, all three outputs are determined. The acquisition functions so far have been for a single output and they need to be modified to account for multiple outputs. The best way to make these modifications is explored.
Three different methods are considered as options to determine the optimal sampling location for the multiple outputs. The first is a round robin method, meaning that the output for which the optimal point is selected is alternated between all the outputs. The number of outputs of the surrogate is denoted as \( k \). Let \( \sigma^2_i \) denote each one of the three different surrogate output variances or epistemic uncertainties where \( i = 1, \ldots, k \). Equation (25) shows the round robin method in which the sampling location is selected by alternating which output is used for the acquisition function over the span of all \( n \) samples.

\[
x^*_i+1 = \arg \min q_{US} (x|\sigma_i) \text{ for } i = j - 1 \mod k + 1.
\]

The second method is the maximum variance method. This approach begins by computing the optimal sampling location \( x'_i \) for each output individually. Next, the GP model is used to predict the epistemic variance at each location \( \sigma^2_i(x'_i) \). This is then normalized by the actual variance of the output data for each output \( \sigma^2_{y,i} \). Lastly, the sampling location with the largest normalized variance is selected because this is the theorized location in which a sample could best reduce the uncertainty of the multiple outputs. Equation (26) denotes the maximum variance method.

\[
x^*_i+1 = \arg \max \left( \frac{\sigma^2_i(x'_i)}{\sigma^2_{y,i}} \right) \text{ for } i = 1, \ldots, k,
\]

where \( x'_i = \arg \min q_{US}(x|\sigma^2_i) \) and \( \sigma^2_{y,i} = \frac{1}{f} \sum_{l=1}^{i} (y_{i,l} - \bar{y}_i)^2 \).

The third method under consideration is called the weighted method. This approach looks for an optimal sampling location by assessing the multiple outputs as a whole rather than individually. Specifically, the statistics of the individual outputs are combined based on a weight factor into a single weighted variance \( \sigma^2_w \) to be used with the US acquisition function. The weight used for each output is the inverse of its training data variance for that given output. This is used as the weight in order to try and normalize the different output variances before they are combined. If they are not normalized, then the variance of one output could dominate the weighted variance, even if it has a low epistemic uncertainty. This could happen because the outputs are not normalized so outputs with larger values would have a larger impact on the weighted variance. Once these individual variances are combined into a single weighted variance, the optimal sampling location is selected using the following acquisition function:

\[
x^*_i+1 = \arg \min q_{US}(x|\sigma^2_w(x)), \quad \text{where } \sigma^2_w(x) = \sum_{i=1}^{k} \frac{\sigma^2_i(x)}{\sigma^2_{y,i}}
\]

An example problem with three outputs \( y_1, y_2, \) and \( y_3 \) is used to evaluate these different multiple output sampling methods. The Park 1, Park 2, and Colville functions listed as equations (28), (29), and (30) are used as the three surrogate outputs [24]. The domain of the input space is \( x_i \in [0,1,1] \) for all \( i = 1, 2, 3, 4 \).

\[
y_1(x) = \frac{2}{3} e^{x_1+x_2} - x_4 \sin (x_3) + x_3
\]

\[
y_2(x) = \frac{x_1}{2} \left[ \sqrt{1 + (x_2 + x_3/x_4)^2} - 1 \right] + (x_1 + 3x_4)e^{1+\sin (x_3)}
\]

\[
y_3(x) = 100\left( x_1^2 - x_2 \right)^2 + (x_1 - 1)^2 + (x_3 - 1)^2 + 90(x_4^2 - x_4)^2
\]
\[+10.1((x_2 - 1)^2 + (x_4 - 1)^2) + 19.8(x_2 - 1)(x_4 - 1)
\]

The various multiple output sampling methods are repeated 100 times to ensure the results are consistent. Figure 8 shows the performance of each sampling method.
Figure 8: Comparison of the round robin, maximum variance, and weighted multiple output active sampling methods for three test cases.

The maximum variance criterion slightly outperforms the other two sampling methods. This is true when looking at each output individually and also when looking at the average MAPE of all three outputs. The only portion of the solution space in which the maximum variance criterion did not outperform the other two methods is in the early stages of the sampling with less than 20 samples for $y_1$. The round robin method allocates optimal samples to this output despite it having the smallest error while the maximum variance method allocates optimal samples for the outputs with larger errors. This allows the round robin method to temporarily outperform the maximum variance method for $y_1$. However, the maximum variance method is able to quickly catch up and outperform the round robin method for this output. The weighted output method underperformed the other two methods. To this end, the maximum variance criterion is selected as the multiple output sampling method for the UUV and submarine hydrodynamic interactions problem.

6 UUV and Submarine Hydrodynamic Interaction

The NMMF active learning GP regression model with the maximum variance multiple output selection criterion is used to model the hydrodynamic interactions between a UUV and submarine. For this study, the low fidelity model is a potential flow solver known as FS-Flow [4]. Low fidelity potential flow solvers use simplified physics which neglects viscosity, skin friction, boundary layer development, flow separation, and leads to the d’Alembert paradox which predicts zero drag on a UUV moving at constant velocity [2]. In order to combat these physical limitations, potential flow solvers are often supplemented with a simple parametric model to help predict the effects of viscosity on the moving body. FS-Flow uses the panel method to resolve the flow around a moving body and supplements the solution with one of four different viscous correlation lines. The frictional resistance can be estimated using the International Towing Tank Conference (ITTC) 57, Hughes, Grigson, or Katsui viscous correlation lines [4]. The ITTC-57 correction line is used for this study.
6.1 Experimental Design

In order for the surrogate to be useful in modeling launch and recovery operations, the surrogate has to accurately predict the hydrodynamic interaction forces and moments on the UUV. For simplicity, only three degrees of freedom of the UUV are considered. This means that the output of the surrogate is chosen to be the well defined surge, $X'$, sway, $Y'$, and yaw, $N'$, coefficients of the UUV [25]. The UUV position, heading angle, speed, and size all have an impact on these output variables, so these parameters are broken down into six input variables.

The UUV position is defined by a longitudinal separation ratio, $R_{Long}$, and a lateral separation ratio, $R_{Lat}$. These parameters are the longitudinal and lateral distance between the centers of buoyancy of the two vehicles normalized by the length of the submarine. The heading angle, $\phi$, is the angle between the body of revolution axes of the two vehicles with a positive heading angle corresponding to the bow of the UUV pointing away from the submarine. Figure 9 and equations (31) and (32) illustrate the longitudinal separation ratio, lateral separation ratio, and heading angle.

$$R_{Long} = \frac{x_{Dist}}{L_{Sub}}$$  \hspace{1cm} (31)

$$R_{Lat} = \frac{y_{Dist}}{L_{Sub}}$$  \hspace{1cm} (32)

Likewise, the UUV length and diameter are non-dimensionalized using ratios. Because the submarine diameter is fixed, the submarine to UUV diameter ratio is selected as a non-dimensional parameter to represent various diameters of the UUV. Lastly, the UUV length to diameter ratio is then used to account for the different lengths of UUV. Table 1 summarizes each of the six different input variables with their accompanying units and bounds.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Units</th>
<th>Bounds</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_{Long}$</td>
<td>Longitudinal Separation Ratio</td>
<td>None</td>
<td>[-0.7,0.7]</td>
</tr>
<tr>
<td>$R_{Lat}$</td>
<td>Lateral Separation Ratio</td>
<td>None</td>
<td>[0.059,0.105]</td>
</tr>
<tr>
<td>$U$</td>
<td>Speed</td>
<td>Knots</td>
<td>[2,5]</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Heading Angle</td>
<td>Degrees</td>
<td>[-2,2]</td>
</tr>
<tr>
<td>$D_{Sub}/D_{UUV}$</td>
<td>Submarine to UUV Diameter Ratio</td>
<td>None</td>
<td>[5,50]</td>
</tr>
<tr>
<td>$L/D_{UUV}$</td>
<td>UUV Length to Diameter Ratio</td>
<td>None</td>
<td>[4.3,13]</td>
</tr>
</tbody>
</table>

As certain bounds are increased, the uncertainty of the high fidelity solutions in some parts of the domain becomes very large. Hence, there is a trade-off between exploration and accuracy. The bounds of this domain are selected to capture the wide range of input variables necessary to simulate UUV motion while being restrictive enough to produce accurate results. While this domain considers this
trade-off, additional constraints are established to exclude certain unrealistic parts of the domain based on input variable interactions. For example, larger diameter UUVs tend to be shorter while smaller diameter UUVs tend to be longer. Two constraints are created based on real-world UUV measurements which ignore the combination of UUV lengths and diameters that are infeasible. Additionally, the relationship between lateral distance from the submarine and the UUV diameter is constrained. This enables the center of buoyancy of the small diameter UUVs to get closer to the submarine. The center of buoyancy of a larger diameter UUV in this region would result in a collision between the vehicles. Likewise, UUVs are constrained, based on diameter, to make sure they are not so far away as to avoid any hydrodynamic interactions.

The NMMF active search algorithm is iterated until 100 high fidelity simulations are performed. At this point, the new high fidelity simulations have little improvement on the accuracy of the surrogate model.

6.2 Computational Fluid Dynamics

The Defense Advanced Research Projects Agency (DARPA) SUBOFF model is used as the geometric model for the submarine hull while the International Submarine Engineering (ISE) Explorer is used for the UUV hull [26, 27]. In accordance with ITTC guidelines, the domain of the CFD simulation has at least one submarine length of clearance between the domain edges and the front of the vehicles. The aft end of the vehicles has three submarine lengths of clearance to the end of the domain [28]. The domain is made symmetrical along the plane that intersects the axes of the two vehicles and a symmetrical boundary layer condition is applied along this plane. This allows the domain to be half the size without compromising accuracy which greatly reduces the computational resources needed for the study.

The CFD simulations use an unstructured polyhedral overset mesh because it more easily accommodates mesh deformation and restructuring as the UUV is re-positioned between simulations [1]. While the number of cells varies between simulations, the number of cells is typically between one and two million which is high enough to provide mesh independence [29].

According to the ITTC, the $k - \epsilon$ and $k - \omega$ turbulence models are by far the most common models that are applied to ship hydrodynamics and have consistently provided accurate predictions [28]. For this simulation, the $k - \omega$ turbulence model is selected because it is more accurate in adverse pressure gradients like those experienced on the stern of the model [28, 30]. The total boundary layer thickness is calculated using the Prandtl’s turbulent boundary layer thickness over a flat plate or $0.16L/Re_L^{1/7}$ [31]. This boundary layer mesh uses prism layers with an expansion ratio of 1.2. The number of prism layers on the submarine and UUV are large enough to ensure that the non-dimensional wall distance $y^+$ values are less than one. This CFD simulation setup is in accordance with ITTC procedures [28].

Once the CFD simulation setup is established, the next step is to validate if the CFD simulations are able to accurately predict real-world results. Submarine and UUV hydrodynamic interactions are very challenging to capture in experimental fluid dynamics (EFD) using tow tank experiments. This is because in order to maintain similitude between vehicle sizes, the UUV models become too small to allow for accurate measurements. However, Leong performed a series of tow tank experiments with a UUV that had a diameter of about 44.7% of the diameter of the submarine [1]. While UUVs of this size are much too large to be considered for actual launch and recovery operations, these real-world experiments are used to validate the CFD setup. Figure 10 compares the surge, sway, and yaw coefficients of the UUV for the CFD and tow tank experiments performed by Leong.
Figure 10: Comparison of the CFD simulated surge, sway, and yaw force and moment coefficients at various longitudinal positions against the experimentally validated tow tank results (EFD) by Leong [1]. All CFD simulated results are within the margin of error of the tow tank results.

Figure 10 shows that the CFD is able to predict all of the surge, sway, and yaw coefficients within the uncertainty of the tow tank experiments in all of the possible UUV locations. The UUV experiences large oscillations in sway and yaw, but the CFD is able to accurately capture these complex real-world hydrodynamic interactions.

6.3 Results

This experimental setup has six input parameters. There is no straightforward way to illustrate a six-dimensional parameter space, but to help visualize the results, a series of two-dimensional plots are generated while keeping the other four input dimensions constant. This allows the impact of each dimension to be assessed while also displaying some interactions between input variables.

Figure 11 shows how the sway and yaw coefficients for the UUV vary as the UUV is in different lateral and longitudinal positions from the submarine. A positive sway coefficient pushes the UUV away from the submarine and a positive yaw moment causes the bow of the UUV to be rotated away from the submarine.
The surge force coefficient also experiences unique hydrodynamic interactions near the bow and stern of the submarine. Figure 12 shows how the surge force coefficient varies along the length of the submarine.
(a) Surge coefficient $X'$ at various $R_{Lat}$ and $R_{Long}$ locations around the submarine at a fixed $U = 3.5$ knots, $\phi = 0^\circ$, $D_{Sub}/D_{UUV} = 20$, and $L/D_{UUV} = 8$. Near the stern of the submarine, the reduced order model predicts a positive surge experienced by the UUV.

(b) Surge coefficient $X'$ at various $D_{Sub}/D_{UUV}$ and $R_{Long}$ locations around the submarine at a fixed $U = 3.5$ knots, $\phi = 0^\circ$, $D_{Sub}/D_{UUV} = 20$, and $L/D_{UUV} = 8$. The reduced order model predicts the positive surge experiences near the stern of the submarine decreases for smaller diameter UUVs.

Figure 12: UUV hydrodynamic interaction surge coefficient $X'$ at various longitudinal separation ratios $R_{Long}$ from the submarine

Figure 12 shows that the UUV experiences a substantial drop in the surge that opposes forward UUV motion near the stern of the submarine at about $R_{Long} \approx -0.4$. In fact, this hydrodynamic interaction causes the surge force to become positive. This means that this hydrodynamic interaction overcomes the drag of the vehicle and would cause it to accelerate forward, even with no thrust from the UUV propeller. There is a low pressure region and the end of the parallel mid-body of the submarine as it transitions to the stern. There is also a high pressure region farther down the stern of the submarine. The UUV experiences this positive surge when it encounters the steep pressure gradient between these two regions. Also, as the fluid flows along the stern of the submarine and passes the bow of the UUV, it accelerates between the two vehicles due to the flow restriction. This Bernoulli effect causes an additional drop in pressure. The drop in pressure near the bow of the UUV also contributes to the drop in surge and causes the UUV to get pulled forward. Additionally, the flow around the stern of the submarine meets the UUV at an angle that causes a large lift. This lift is perpendicular to the flow and causes the resultant total force vector to have a component in the forward direction of the UUV. Figure 13 summarizes this complex interaction.
This specific hydrodynamic interaction could make UUV launch and recovery operations particularly challenging because it is so far outside of the normal UUV operating window. Figure 12b shows that the magnitude of this hydrodynamic interaction decreases as the diameter of the UUV decreases. This means that smaller UUVs would be better equipped to overcome this hydrodynamic interaction.

A long slender body without control fins experiences a destabilizing effect when in steady translation. This is known as the munk moment [2]. Also, the sway force increases as the heading angle increases, just like lift increases when the angle of attack increases on an airfoil. Figure 14 shows how the sway and yaw coefficients vary based on the heading angle and speed of the UUV.

Figure 14: Maps of the sway and yaw coefficients $Y'$ and $N'$ at various heading angles $\phi$ and speeds $U$ for a UUV with a fixed $R_{Long} = 0$, $R_{Lat} = 0.08$, $D_{Sub}/D_{UUV} = 20$, and $L/D_{UUV} = 8$. The reduced order model predicts a near-linear relationship between the sway and heading angle.

Figure 14 shows that if the vehicle is at a non-zero heading angle, there is a resulting moment that
will cause the vehicle to rotate in the direction in which it is angled. The surrogate model is able to accurately predict the munk moment of the UUV and the expected behavior of how it increases nearly linearly with heading angle [3]. The same relationship between heading angle and sway exists and the reduced order model accurately captures this phenomenon as well. Additionally, because the sway and yaw coefficients are non-dimensionalized using the velocity of the vehicle, these non-dimensional coefficients are expected to be relatively independent of the speed [2, 3]. The surrogate model is also able to capture this effect.

The total drag on a submarine or UUV is a combination of its pressure (form) drag, caused by wake formation and boundary layer separation, and its viscous (skin) drag, caused by the fluid friction on the wetted surface of the vehicle. Vehicles have an optimal length to diameter ratio that reduces the drag on the vehicle [32]. When vehicles are very short, they are more like bluff bodies and have a lot of pressure drag. As the vehicle becomes longer, the form drag decreases. However, when they become too long, the increase in viscous drag outweighs the loss in pressure drag. This means that a UUV has an optimal length to diameter ratio \( L/D_{UUV} \) for reducing drag. The optimal \( L/D_{UUV} \) is dependent on the shape and speed of the vehicle and is usually in the range of 8 to 12. Figure 15 shows how the surge coefficient varies with \( L/D_{UUV} \).

This illustrates how the surrogate model is able to identify the trade-off between pressure drag and viscous drag for different UUV lengths. The surrogate identified the optimal length to diameter ratio of the UUV to be between about 10 and 12 for the given UUV shape and speed. Additionally, the surrogate also found that the surge is relatively independent of small changes in heading angles. This is consistent with real world results [33]. The gray region in figure 15a represents a constraint within the domain that is unexplored due to UUV infeasibility.

Another benefit of the NMMF active learning GP regression process is that the process models the difference between the high fidelity (CFD) and low fidelity (potential flow) simulations. This means that the results of the model can be used to identify scenarios in which the low fidelity simulations fail to capture the accuracy of the high fidelity simulations. Figure 16 compares the sway and yaw coefficients for the high fidelity surrogate model with the low fidelity model surrogate.
(a) High fidelity surrogate model prediction of the sway coefficient $Y'$ of a UUV at various $R_{Lat}$ and $R_{Long}$ for a fixed $U = 3.5$ knots, $\phi = 0^\circ$, $D_{Sub}/D_{UUV} = 20$ and $/D_{UUV} = 8$.

(b) High fidelity surrogate model prediction of the yaw coefficient $N'$ of a UUV at various $R_{Lat}$ and $R_{Long}$ for a fixed $U = 3.5$ knots, $\phi = 0^\circ$, $D_{Sub}/D_{UUV} = 20$ and $/D_{UUV} = 8$.

(c) Low fidelity surrogate model prediction of the sway coefficient $Y'$ of a UUV at various $R_{Lat}$ and $R_{Long}$ for a fixed $U = 3.5$ knots, $\phi = 0^\circ$, $D_{Sub}/D_{UUV} = 20$ and $/D_{UUV} = 8$.

(d) Low fidelity surrogate model prediction of the yaw coefficient $N'$ of a UUV at various $R_{Lat}$ and $R_{Long}$ for a fixed $U = 3.5$ knots, $\phi = 0^\circ$, $D_{Sub}/D_{UUV} = 20$ and $/D_{UUV} = 8$.

Figure 16: Comparison of sway and yaw coefficients of the high and low fidelity surrogate models of a UUV at various $R_{Lat}$ and $R_{Long}$ for a fixed $U = 3.5$ knots, $\phi = 0^\circ$, $D_{Sub}/D_{UUV} = 20$ and $/D_{UUV} = 8$. Low fidelity cannot accurately predict sway coefficient $Y'$, but performs well for yaw coefficient $N'$.

Figure 16 shows that the low fidelity potential flow model is quite accurate at predicting the yaw coefficient, but has major limitations on predicting the sway coefficient due to the d’Alembert paradox [2, 3]. Another example of this limitation is the inability of the low fidelity potential flow solver to determine the change in sway at various heading angles. Figure 17 compares heading angles from the potential flow model with the high fidelity CFD model. This shows how potential flow is not able to accurately capture how the sway coefficient varies with changes to the heading angle. In fact, the low fidelity model predicts an almost constant near zero value which is about the midpoint of the actual CFD results. This is the best that the low fidelity model can do because of the d’Alembert paradox of potential flow.
(a) High fidelity surrogate model prediction of the sway coefficient $Y'$ of a UUV at various heading angles $\phi$ and speeds $U$ with a fixed $R_{\text{Long}} = 0$, $R_{\text{Lat}} = 0.08$, $D_{\text{Sub}}/D_{\text{UUV}} = 20$, and $L/D_{\text{UUV}} = 8$. This accurately reflects the near-linear relationship between sway and heading angle.

(b) Low fidelity surrogate model prediction of the sway coefficient $Y'$ of a UUV at various heading angles $\phi$ and speeds $U$ with a fixed $R_{\text{Long}} = 0$, $R_{\text{Lat}} = 0.08$, $D_{\text{Sub}}/D_{\text{UUV}} = 20$, and $L/D_{\text{UUV}} = 8$. This predicts a constant near-zero value as expected by the d'Alembert paradox.

Figure 17: Comparison of sway and yaw coefficients of the high and low fidelity surrogate models of a UUV at various heading angles $\phi$ and speeds $U$ with a fixed $R_{\text{Long}} = 0$, $R_{\text{Lat}} = 0.08$, $D_{\text{Sub}}/D_{\text{UUV}} = 20$, and $L/D_{\text{UUV}} = 8$. Low fidelity cannot accurately predict sway coefficient $Y'$ due to the d'Alembert paradox.

Because the d'Alembert paradox is a known limitation of potential solvers, many of them have built-in parametric models that augment the results. The d'Alembert paradox results in potential flow predicting zero drag around the UUV in steady state translation. In order to provide more accurate results, the FS-Flow potential flow solver uses the ITTC-57 parametric equations to estimate the drag of the UUV [4]. Figure 18 compares the surge coefficients of the high fidelity CFD surrogate model and the low fidelity potential flow surrogate model at various UUV length to diameter ratios.

(a) High fidelity surrogate model prediction of the surge coefficient $X'$ of a UUV at various $L/D_{\text{UUV}}$ and heading angles $\phi$ with a fixed $R_{\text{Long}} = 0$, $R_{\text{Lat}} = 0.08$, $U = 3.5$ knots, and $D_{\text{Sub}}/D_{\text{UUV}} = 20$.

(b) Low fidelity surrogate model prediction of the surge coefficient $X'$ of a UUV at various $L/D_{\text{UUV}}$ and heading angles $\phi$ with a fixed $R_{\text{Long}} = 0$, $R_{\text{Lat}} = 0.08$, $U = 3.5$ knots, and $D_{\text{Sub}}/D_{\text{UUV}} = 20$. The ITTC-57 parametric equations of the low fidelity potential flow simulations are similar to the results of the high fidelity CFD simulations.

Figure 18: Comparison of surge coefficient $X'$ of the high and low fidelity surrogate models of a UUV at various $L/D_{\text{UUV}}$ and heading angles $\phi$ with a fixed $R_{\text{Long}} = 0$, $R_{\text{Lat}} = 0.08$, $U = 3.5$ knots, and $D_{\text{Sub}}/D_{\text{UUV}} = 20$. The ITTC-57 parametric equations of the low fidelity potential flow simulations are similar to the results of the high fidelity CFD simulations.

Overall, the built-in ITTC-57 parametric equations of the low fidelity potential flow model are very similar to the model trained on CFD data. This allows the potential flow model to predict the surge...
coefficient much better than the sway coefficient.

In order to test the accuracy of the model, a total of 500 Latin hypercube samples are used as test data for the GP regression model. The mean absolute error (MAE) between the predicted and actual results of the test data is determined for each of the three outputs. Because these values are hard to conceptualize, MAE can be thought of in terms of the control system of the UUV and how these errors translate into UUV control values. The MAE of the sway and yaw coefficients can be represented as an equivalent rudder angle difference \( \Delta \delta_{eq,Y} \) or \( \Delta \delta_{eq,N} \). This is how much the rudder angle would need to change to produce the force or moment equivalent to the MAE. Likewise, the surge coefficient MAE can be thought of as a percentage of the propulsive force of the UUV (% \( X_{prop} \)). The hydrodynamic coefficients in use to determine these error equivalents are taken from the Remus 100 [34].

<table>
<thead>
<tr>
<th>Output</th>
<th>Mean Absolute Error</th>
<th>Error Equivalents</th>
</tr>
</thead>
<tbody>
<tr>
<td>( X' )</td>
<td>7.492E-04</td>
<td>10.26% ( X_{prop} )</td>
</tr>
<tr>
<td>( Y' )</td>
<td>7.220E-04</td>
<td>( \Delta \delta_{eq,Y} = 1.78 ) degrees</td>
</tr>
<tr>
<td>( N' )</td>
<td>7.629E-04</td>
<td>( \Delta \delta_{eq,N} = 3.95 ) degrees</td>
</tr>
</tbody>
</table>

Overall, the reduced order model is able to predict the complex surge, sway, and yaw hydrodynamic interactions that are determined using the CFD simulations and validated against tow tank experiments. This reduced order model can be used to determine the hydrodynamic interactions in real time, which is several orders of magnitude faster than performing a CFD simulation, which usually takes several hours to complete.

## 7 Conclusions

We have formulated a new method for active sampling that is non-myopic and also utilizes models of multiple fidelity. The new approach allows for efficiently computing reduced-order models with unprecedented accuracy due to its non-myopic active search properties. It is ideal for situations where plentiful and accurate training data is not easy to obtain, e.g. because of high computational cost. We have first demonstrated the advantages of the new approach in two representative prototype systems, as well as a more realistic setup involving the hydrodynamic interactions between a UUV and a submarine.

The non-myopic multi-fidelity active learning GP regression surrogate model is able to accurately predict the complex hydrodynamic interactions between a submarine and UUV. Specifically, the obtained model is able to mimic the high accuracy of the CFD while being able to predict these hydrodynamic interactions in real-time. Real-time modeling of these hydrodynamic interactions is essential to simulate the motion required to launch and recover UUVs from submarines. To this end, this surrogate model may be integrated into UUV control and autonomy systems and motion simulators to predict these hydrodynamic interactions and further enable UUV launch and recovery from submarines. We leave this direction as future work.

Other future directions may include increasing the dimensionality of the input space by considering more parameters, i.e. more complex and realistic setups. Such a step would require more effective surrogate models such as those based on neural networks or operators [35]. This could include introducing movable control surfaces on the UUV, modeling transient UUV behavior, accounting for six degrees of freedom motion by allowing the two vehicle axes to be non-planar, or simulating the hydrodynamic interactions near a submarine appendage like the sail or a dry deck shelter. In addition, the application of this non-myopic multi-fidelity sampling algorithm is currently formulated for only two levels of fidelity. An extension to more levels of fidelity is straightforward and may be beneficial for other CFD problems, e.g. by using variable resolution CFD models. This is also left to be explored in future work.
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References


