Design, Evaluation, and Validation of a Naval Ship Structural Health Monitoring Tool

by

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B.S., United States Naval Academy (2010)

Submitted to the Department of Mechanical Engineering in partial fulfillment of the requirements for the degrees of Naval Engineer and Master of Science in Mechanical Engineering at the MASSACHUSETTS INSTITUTE OF TECHNOLOGY June 2017

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Abstract

The US Navy implements structural preventative maintenance procedures onboard its vessels using ship's personnel inspection. These procedures have been largely successful in identifying major problem areas before any interference with mission execution has occurred. However, changes in the Navy's manning philosophy to minimal manning and new ship designs focused on automation encourage a re-evaluation of these structural preventative maintenance procedures. Automation of structural inspection and damage detection would reduce associated manpower costs as well as inform better preventative maintenance schedules for US Navy vessels.

This study outlines a modeling tool for structural health monitoring using non-linear Kalman Filter methodologies such as the Extended Kalman Filter and the Ensemble Kalman Filter to identify damage within a structural model. Through the observation of structural responses and the formulation of a Kalman Filter, it is possible to produce estimates of structural parameters related to damage, specifically changes to elastic modulus and changes in material density. The results of this modeling tool were evaluated to quantify the time and length scales required for damage detection and were validated against a structural model generated in the MAESTRO Global Structural Analysis software suite.

Thesis Supervisor: Themistoklis P. Sapsis
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Contents

1 Introduction 17
   1.1 Structural Health Monitoring in Naval Vessels 17
   1.2 Kalman Filter Methodology 19
   1.3 Research Focus 21

2 Structural Damage Identification 23
   2.1 Homogeneous Problem Formulation 23
      2.1.1 Governing ODE 23
      2.1.2 Extended Kalman Filter Formulation: Homogeneous Case 26
   2.2 Inhomogeneous Problem Formulation 34
      2.2.1 Governing ODE 34
      2.2.2 Extended Kalman Filter Formulation: Inhomogeneous Case 35
   2.3 Extended Kalman Filter Model Limitations 43

3 Nonlinear Damage Identification 45
   3.1 Problem Formulation 45
      3.1.1 Governing ODE 46
   3.2 Ensemble Kalman Filter Formulation 49
      3.2.1 Ensemble Member Update 50
      3.2.2 Noise Parameter Optimization 51
      3.2.3 Ensemble Kalman Filter Results 52
   3.3 Ensemble Kalman Filter Model Limitations 54

7
4 Scenario-Driven Damage Identification

4.1 Problem Formulation

4.1.1 Governing ODE

4.1.2 Higher Modes Resolution: Rescaling

4.2 Ensemble Kalman Filter: Single Area of Damage

4.2.1 Ensemble Member Update

4.2.2 Noise Parameter Optimization

4.2.3 Ensemble Kalman Filter Results

4.3 Ensemble Kalman Filter: Multiple Damage Areas

4.4 Scenario-Driven Damage Identification Resolution

5 Validation

5.1 MAESTRO Finite Element Model

5.1.1 Time-Domain Analysis Results

5.2 Governing Equations and Formulation

5.2.1 Higher Modes Resolution: Rescaling

5.2.2 Kalman Filter Application and Noise Parameter Optimization

5.3 Damage Implementation

5.4 Validation Results

5.4.1 Undamaged Scenario

5.4.2 Damaged Bow

5.4.3 Damaged Midbody

5.5 Discussion of Results

6 Conclusion and Recommendations

6.1 Conclusions

6.2 Suggestions for Future Work

A Multiple Damage Areas

A.1 ODE Solver Function

A.2 Ensemble Kalman Filter
List of Figures

2-1 Change in Deflection at Mode $N = 1$ ........................................ 28
2-2 Change in Deflection at Mode $N = 2$ ........................................ 29
2-3 Change in Deflection at Mode $N = 3$ ........................................ 29
2-4 Change in Deflection at Mode $N = 4$ ........................................ 30
2-5 Change in Deflection at Mode $N = 5$ ........................................ 30
2-6 Change in Beam Parameter $\theta_1$ at Mode $N = 1$ ......................... 32
2-7 Change in Beam Parameter $\theta_1$ at Mode $N = 1$ ......................... 33
2-8 Beam Profile Definitions ........................................................... 34
2-9 Modal Coefficient for $n = 1$, Measuring Mode $N = 1$ Modes ........... 37
2-10 Modal Coefficient for $n = 2$, Measuring Mode $N = 2$ Modes .......... 38
2-11 Modal Coefficient for $n = 3$, Measuring Mode $N = 3$ Modes .......... 38
2-12 Beam Parameter $\theta$ with $N = 1$ Modes Measured ........................ 39
2-13 Beam Parameter $\theta$ with $N = 2$ Modes Measured ........................ 40
2-14 Beam Parameter $\theta$ with $N = 3$ Modes Measured ........................ 40
2-15 Beam Parameter $\theta$ with $N = 1$ Modes Measured in Space-Time Domains 41
2-16 Beam Parameter $\theta$ with $N = 2$ Modes Measured in Space-Time Domains 42
2-17 Beam Parameter $\theta$ with $N = 3$ Modes Measured in Space-Time Domains 42
3-1 Modal Response for $n = 5$, Measuring $N = 5$ Modes ...................... 48
3-2 Material Response for $n = 1$, Measuring $N = 2$ Modes ................... 52
3-3 Material Response for $n = 2$, Measuring $N = 2$ Modes ................... 52
3-4 Material Response for $n = 1$, Measuring $N = 3$ Modes ................... 53
3-5 Material Response for $n = 2$, Measuring $N = 3$ Modes ................... 53
3-6 Material Response for n = 3, Measuring N = 3 Modes

4-1 Forward Structural Damage on CG-47 Class

4-2 Superstructure Structural Damage on CG-47 Class

4-3 Modal Response for n = 3, Measuring N = 3 Modes

4-4 Position Response for n = 2, Measuring N = 2 Modes

4-5 Damage Response for n = 2, Measuring N = 2 Modes

4-6 Position Response for n = 3, Measuring N = 3 Modes

4-7 Damage Response for n = 3, Measuring N = 3 Modes

4-8 Modal Response for n = 2, Measuring N = 2 Modes

4-9 Position Response for n = 2, Measuring N = 2 Modes

4-10 Damage Response for n = 2, Measuring N = 2 Modes

4-11 Position Response for n = 3, Measuring N = 3 Modes

4-12 Damage Response for n = 3, Measuring N = 3 Modes

5-1 MAESTRO Model Perspective

5-2 MAESTRO Model Profile

5-3 Model Shear Force

5-4 Vertical Shear Force of Undamaged OSV

5-5 Vertical Bending Moment of Undamaged OSV

5-6 Modal Deflection of Undamaged OSV

5-7 Forcing of Undamaged OSV

5-8 Damage Implementation to Stiffness Material Property

5-9 Undamaged Scenario Modal Position

5-10 Undamaged Scenario Damage Intensity, \( y_1 = 0 \)

5-11 Undamaged Scenario Damage Intensity, \( y_1 = 0.1 \)

5-12 Modal Deflection with Damaged Bow

5-13 Modal Forcing with Damaged Bow

5-14 Position Response for n = 3, Measuring N = 3 Modes

5-15 Damage Response for n = 3, Measuring N = 3 Modes

5-16 Modal Deflection with Damaged Midbody
<table>
<thead>
<tr>
<th>Page</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>5-17</td>
<td>Modal Forcing with Damaged Midbody</td>
<td>95</td>
</tr>
<tr>
<td>5-18</td>
<td>Position Response for $n = 3$, Measuring $N = 3$ Modes</td>
<td>95</td>
</tr>
<tr>
<td>5-19</td>
<td>Damage Response for $n = 3$, Measuring $N = 3$ Modes</td>
<td>96</td>
</tr>
</tbody>
</table>
# List of Tables

3.1 HSV-2 SWIFT Characteristics ............................................. 46  
3.2 Nonlinear Damage Identification Noise Parameters ................. 51  
4.1 Single Area of Damage: Noise Parameters .............................. 64  
4.2 Multiple Areas of Damage: Noise Parameters ......................... 68  
5.1 MAESTRO Model Principal Dimensions and Material Properties ... 75  
5.2 Validation Noise Parameters ............................................. 86  
5.3 Damage Implementation ................................................... 87
Chapter 1

Introduction

1.1 Structural Health Monitoring in Naval Vessels

A ship is a structure subject to an uncertain environment with uncertain load conditions for an uncertain amount of time. Under such circumstances, risk mitigation through the quantification and categorization of the structural performance of a ship is necessary to ensure structural longevity and the vessel’s safe operation. This process can be achieved through various strategies that take advantage of the proliferation of cheap and efficient sensors to identify stresses and damage points resulting from dynamic load conditions. These various strategies for damage identification are a subset of the Structural Health Monitoring (SHM) field of study.

SHM for naval application should take advantage of naval architects’ understanding of load conditions and structural effects on a vessel. Various loading conditions can be categorized as static, low-frequency dynamic, high-frequency dynamic, impact, or operational ([23], [29], [20], [18]). A brief description of each of these loads is included here:

1. Static loading will be present from differences in a ship’s buoyant forces and the weight of the ship. These “still-water loads” will contribute to shear forces and bending moments in the structure. If a ship does not undergo major structural changes over its lifetime, this loading condition may remain relatively constant.
2. Low-frequency dynamic loading will result from wave-induced forces on the hull. Waves will have a pressure component acting on the hull and as a ship moves through these waves, accelerations from this movement will impart an inertial response to the structure. This loading condition is a major contributor to structural fatigue (an accumulative damage effect from cyclic loading).

3. High-frequency dynamic loading occurs when a vibratory response is imparted to the structure; one such example is unbalanced rotating machinery transmitting forced vibrations into the hull.

4. Impact loading may be imparted from vibratory stress present in a “slamming” event. “Slamming” is where the “bow emerges from one wave crest only to re-enter with significant impact into the next wave” [23]. This action can impart nonlinear, high frequency vibrations that have a significant local impact load.

5. Operational loading for ships may include sloshing forces within partially-filled tanks. During the maintenance cycle, structural loads will be imparted from ship launching and drydocking. In the unforeseen situation of groundings or collisions, additional structural effects will be seen. Combatant naval vessel structures will also be subjected to weapons-effects in a post-military attack [29].

Vessel loading conditions provide an informed starting point for the identification of structural fatigue and areas of local damage. This starting point has been incorporated into the thesis research presented in Chapter 4, Scenario-Driven Damage Identification.

Current Work in the Field

With these various sea loading conditions on vessels, current work in SHM seeks to identify the accumulative damage effect of loads on the structure, or the fatigue on a ship. Significant amounts of work have been done in this area by various organizations
including academics and the commercial shipping industry, as well as the American ([21], [26]), Australian [23], French, and Norwegian [8] navies.

Various approaches to fatigue quantification have utilized traditional fatigue calculations such as Miner’s hypothesis and S-N curves applied to different sensor data sets such as impedance, strain, acceleration, or acoustic noise [24] associated with the structure. Other work in this area has sought to quantify the effect of different sea loadings on the structure using sparse data sets [7]. In almost all cases, the goal is the quantification of structural response to determine the vessel’s structural lifespan.

As a complementary approach to damage identification present in fatigue analysis, a Kalman Filter methodology may be used to identify, update, and localize the amount of damage present on a ship. If done in support of long-term and continuous SHM, the data returned from the Kalman Filter can aid in validation of fatigue life models. Research presented in this thesis attempts to quantify structural uncertainties present in naval vessels through such a damage identification method.

1.2 Kalman Filter Methodology

The approach to damage identification will be done through a data assimilation method. It is assumed that the current (a priori) state of the ship’s structure (system) is a known quantity. By using sensor data as a set of observations, the state of the system can be estimated at a given time. Deviations from the a priori state indicate structural deterioration and possible damage points within the ship’s structure.

The Kalman Filter (KF) is a sequential data assimilation algorithm developed by Rudolf E. Kálmán that estimates a new state using time-stepped measurements with statistical noise [16]. As this data assimilation process is conducted sequentially in time, the uncertainty in the estimated state must be evolved from one observation point to the next. As a result, this algorithm works in a two-step process that includes a prediction step and an observation update step ([13],[9],[11]). The prediction step
will evolve the mean state and error covariance to time of observation:

\[ x_{k+1} = F x_k \]

\[ P_{k+1} = FP_k F^T + Q \]

where \( Q \) is the error covariance matrix which allows for the model errors.

The observation update step then updates both the state and the covariance given a known state, \( x_k \), and the a priori uncertainty, \( P_{k|k-1} \):

\[ x_{k|k} = x_k + P_{k|k-1} H^T (HP_{k|k-1} H^T + R)^{-1} (y - H x_k) \]

\[ P_{k|k} = P_{k|k-1} + P_{k|k-1} H^T (HP_{k|k-1} H^T + R)^{-1} H P_{k|k-1} \]

where \( H \) is a measurement operator and \( R \) is statistical noise present in measurement covariances (in the KF, this is assumed to be Gaussian). The \( y \) term is the observation vector and will relate the measurement operator to the true state \( (x^{true}) \) with the inclusion of some measurement errors \( (v) \). This relationship is expressed as \( y = H x^{true} + v \). In short-hand, the \( P_{k|k-1} H^T (HP_{k|k-1} H^T + R)^{-1} \) term is set equal to \( K \) and is known as the Kalman Gain Factor. It is important to note that the KF assumes that the relationship between this observation operator and the evolution model is linear. As a result, the KF works well for linear problems.

The Extended Kalman Filter (ExKF) builds off the KF as an algorithm for the estimations of new states. However, the ExKF has been adapted for use with nonlinear models. For a nonlinear model:

\[ x_{k+1} = f(x_k) \]

the error covariance equation remains the same, but \( F \) is the Jacobian of \( f(x_k) \). This linearization is used for the prediction of error statistics used in calculating the mean state ([6],[11]).
The Ensemble Kalman Filter (EnKF) evolves the ExKF to explicitly address highly-nonlinear models through the use of Monte Carlo simulation techniques. Developed by Geir Evensen in 1994, it takes advantage of an ensemble of representative model states to propagate the mean state and covariance forward in time ([9],[12]). Its application in this research is discussed in Chapters 3 to 5.

1.3 Research Focus

This thesis uses nonlinear Kalman Filter methodology to identify, update, and localize the amount of damage present on a ship. Chapter 2 applies the Extended Kalman Filter discussed in Section 1.2 to an Euler-Bernoulli beam with both homogeneous and inhomogeneous material properties for initial model formulation. Due to limitations of the Extended Kalman Filter in addressing the problem of ship structural damage identification, the model was revised to incorporate an Ensemble Kalman Filter approach. Chapter 3 discusses the Ensemble Kalman Filter as applied to the entire vessel. Chapter 4 uses an understanding of sea loading to tailor the Ensemble Kalman Filter as part of a scenario-driven identification scheme. A validation model based on this tailored Ensemble Kalman Filter approach is presented in Chapter 5 with concluding statements in Chapter 6. Code used to generate final results in Chapter 4 and 5 are included in the appendices.
Chapter 2

Structural Damage Identification

2.1 Homogeneous Problem Formulation

Due to a naval vessel’s similarity to a beam, an initial problem was formulated to identify areas of damage in a simply-supported Euler-Bernoulli beam. Using an Extended Kalman Filter model, this approach defines a simply-supported beam, conversion to state-space model, and identification of beam modes prior to identifying differences within the material properties. This model measures damage by identifying significant changes in material properties of the beam, either stiffness \((EI/\mu)\) or damping \((c/\mu)\). These structural parameters are first identified in the a priori condition before being changed as part of model validation.

2.1.1 Governing ODE

Equation 2.1 is the governing, fourth-order partial differential equation for an Euler-Bernoulli beam of length \(L\).

\[
\mu(x)\left(\frac{d^2w(t,x)}{dt^2}\right) + c\left(\frac{dw(t,x)}{dt}\right) + E(x)I(x)\left(\frac{d^4w(t,x)}{dx^4}\right) = q(t,x)
\]  

(2.1)

where \(w(t,x)\) is the deflection of the beam at time \(t\) and location \(x\)

\(q(t,x)\) is the transverse excitation at time \(t\) and location \(x\)
\( \mu(x) \) is the mass per unit length at location \( x \)

\( c \) is the damping coefficient

\( E(x) \) is the elastic modulus at location \( x \)

\( I(x) \) is the second moment of area at location \( x \).

The initial assumptions for this problem are that the beam is homogeneous, there is no damping \( (c = 0) \), and the beam is in free vibration \( (q(t, x) = 0) \). Simply-supported boundary conditions were applied which set the deflection and moment equal to zero at the origin and endpoint of the beam

\[
\begin{align*}
w(t, x) & \bigg|_{x=0,L} = 0 \\
\frac{d^2w(t, x)}{dt^2} & \bigg|_{x=0, L} = 0
\end{align*}
\]

Using a separation of variables approach, the deflection can be broken down into both time and spatial components

\[
w_n(t, x) = f_n(t)g_n(x)
\]

\[
w_n(t, x) = \sum_{n=1}^{\infty} C_n w_n(t, x) = \sum_{n=1}^{\infty} C_n f_n(t) \sum_{n=1}^{\infty} C_n g_n(x)
\]

where \( C_n \) is representative of arbitrary constants.

When the spatial functions have been separated from the temporal functions, the spatial functions are solved for using trigonometric functions where

\[
g_n(x) = A_1 \cos(\beta x) + A_2 \sin(\beta x) + A_3 \cosh(\beta x) + A_4 \sinh(\beta x)
\]

Solving for a non-trivial solution to \( g_n(x) \) yields

\[
g_n(x) = \sin\left(\frac{\pi nx}{L}\right)
\]
Substituting the simply-supported boundary conditions back into the governing equation yields

\[
\sum_{n=1}^{\infty} \left[ \mu(x)(f''_n(t)g_n(x)) + c(f'_n(t)g_n(x)) + E(x)I(x)(f_n(t)\frac{d^4g_n(x)}{dx^4}) \right] = 0
\]

where \( f'_n(t) \) is representative of the time derivative.

Since the deflection function \( w_n(t, x) \) is an approximate solution only, a trial function of the same form as \( g_n(x) \) must be introduced to maintain orthogonality and leads to the form

\[
f''_n(t) \int_0^L \mu(x)g_m(x)g_n(x)dx + cf'_n(t) \int_0^L g_m(x)g_n(x)dx + f_n(t) \int_0^L E(x)I(x)g_m(x)\frac{d^4g_n(x)}{dx^4}dx = 0
\]

where \( \int_0^L \mu(x)g_m(x)g_n(x)dx \) is the mass matrix, \( M_{mn} \)

\( \int_0^L g_m(x)g_n(x)dx \) is the damping matrix, \( J_{mn} \)

\( \int_0^L E(x)I(x)g_m(x)\frac{d^4g_n(x)}{dx^4}dx \) is the stiffness matrix, \( K_{mn} \).

In the case where the material properties \( \mu(x) \), \( E(x) \), and \( I(x) \) are constant in space (i.e., \( \mu(x) = \mu_o \) and \( E(x)I(x) = E_oI_o \)) and the spatial terms \( g_n(x) \) and \( g_m(x) \) are defined as \( g_n(x) = \sin(\frac{\pi x}{L}) \) and \( g_m(x) = \sin(\frac{\pi m x}{L}) \), these matrices simplify to

\[
M_{mn} = \frac{1}{2} I_{mn}\mu_o
\]

\[
J_{mn} = \frac{1}{2} I_{mn}
\]

\[
K_{mn} = \frac{1}{2} \left( \frac{\pi^4n^4}{L^4} \right) E_oI_o
\]

As a result, the governing equation in the event of free vibration is simplified to

\[
\sum_{n=1}^{N} \left[ f''_n(t)\left( \frac{\mu_o}{2} \right) + f'_n(t)\left( \frac{C}{2} \right) + \frac{1}{2} \left( \frac{\pi^4n^4}{L^4} \right) E_oI_o \right] = 0 \quad (2.2)
\]

where \( N \) is the number of modes that will be kept.
2.1.2 Extended Kalman Filter Formulation: Homogeneous Case

Extended Kalman Filter for Modal Response

As described in the beginning of this chapter, the Extended Kalman Filter model was set up to identify damage through variations of beam parameters \( \frac{E_o I_o}{\mu_o} \) and \( \frac{c}{\mu_o} \). Before solving for these parameters directly, it was necessary to estimate the current state of beam deflection and velocity for \( n = N \) desired modes. The state of this modal response was represented by 

\[
F_n(t) = \begin{bmatrix} f_n(t) \\ f_n'(t) \end{bmatrix}
\]

Using the free vibration case described in Equation 2.2, the acceleration term \( (f''_n(t)) \) was solved for as

\[
f''_n(t) = -f'_n(t) \frac{c}{\mu_o} - f_n(t) \left[ \frac{E_o I_o}{\mu_o} \left( \frac{\pi^4 n^4}{L^4} \right) \right]
\]

In order to take advantage of a state-space representation approach, a first-order model was written by setting \( f'_n(t) = h_n \). This first-order model was

\[
\begin{align*}
  f'_n(t) &= h_n(t) \\
  h'_n(t) &= -h_n(t) \frac{c}{\mu_o} - f_n(t) \frac{E_o I_o}{\mu_o} \left( \frac{\pi^4 n^4}{L^4} \right)
\end{align*}
\]

Using a linear system in standard state-space form the first-order model was then approximated to

\[
\begin{cases}
  F'_n(t) = A\vec{x} + B(\vec{u} + \vec{w}) \\
  \vec{y} = C\vec{x} + \vec{v}
\end{cases}
\]

where \( \vec{x} \) will be set equal to \( F_n(t) \) to approximate the beam deflection and velocity. \( A \) represents the state dynamics of the model and is updated at each modal iteration (each \( n \)) to include all previous modes. \( A \) is defined as

\[
A = \begin{bmatrix}
  0 & I_{mn} \\
  -\frac{E_o I_o}{\mu_o} \frac{\pi^4 n^4}{L^4} I_{mn} & -\frac{c}{\mu_o} I_{mn}
\end{bmatrix}
\]

\( I_{mn} \) is an identity matrix to represent the spatial functions and has dimension of \( 2 \times N \). \( B \) describes the input gain matrix for the control signal. The control signal,
\( \ddot{u} \), is defined as a random oscillating time-dependent vector,

\[
\ddot{u} = \sin(t/5) + 0.3\sin(t/10) + 0.2\sin(t/3) + 0.5\sin(t/7)
\]

The observation vector, \( \ddot{y} \), is comprised mainly of a linear combination of states where \( C \) is an assumed measurement sensitivity matrix. Some Gaussian noise, \( \ddot{u} \) and \( \ddot{v} \), is added to the system and is assumed to closely approximate the acceleration of the beam (assumed to be sinusoidal).

The Extended Kalman Filter is used to identify the \( F(t) \) states for \( N = 5 \) modes for a time scale of approximately 60 seconds. The initial estimates for the state and covariance were assumed as

\[
\hat{x} = [0; 0] \\
P = BQB' = [0; 1]Q[0, 1]
\]

Measurement sensitivity \( C \), measurement noise \( Q \), and process noise \( R \) were assumed as

\[
C = [1; 0] \\
Q = R = 0.01
\]

The Kalman Gain factor \( (K_k) \) was calculated as

\[
K_k = P_{k|k-1}C_k^T \times \left[ C_kP_{k|k-1}C_k^T + R_k \right]^{-1}
\]

The algorithm used to determine the modal coefficients was then computed using a prediction and an update step. The update step was defined as

\[
\dot{X}_{k|k} = F_k \ddot{X}_k + K_k \left\{ \ddot{y}_k - C_k \ddot{X}_{k|k-1} \right\}
\]

\[
P_{k|k} = P_{k|k-1} - K_k C_k P_{k|k-1}
\]
The prediction step was defined as

\[
\hat{X}_{k+1|k} = A \hat{X}_{k|k-1} + Bu
\]

\[
P_{k+1|k} = A_k P_{k|k} A_k^T + B Q_k B'
\]

The resulting estimations for five modal deflections are shown in Figures 2-1 to 2-5. The modal response is cumulative and the algorithm shows an improvement in error with increased mode \(n\).

Figure 2-1: Change in Deflection at Mode \(N = 1\)
Figure 2-2: Change in Deflection at Mode N = 2

Figure 2-3: Change in Deflection at Mode N = 3
Figure 2-4: Change in Deflection at Mode N = 4

Figure 2-5: Change in Deflection at Mode N = 5
Extended Kalman Filter for Material Parameter Response

Having solved for the modal response, the Extended Kalman Filter was used in a second iteration to solve for variations in the beam material parameters. Using the same state-space form and prediction-update algorithm presented in Section 2.1.2, the first-order model’s variables were updated to approximate material parameter response. Setting the stiffness material property \( \frac{E_o I_o}{\mu_o} = r \), the first-order model was defined as

\[
\begin{align*}
\dot{r}_n' &= 0 \\
\dot{h}_n'(t) &= -h_n(t) \frac{E_o I_o}{\mu_o} - f_n(t) r_n (\pi^2 n^2 L^2)
\end{align*}
\]

The state dynamics of this model are represented as a matrix, \( A_{\text{new}} \), given state

\[
R = [r_n, r_n']^T
\]

\[
A_{\text{new}} = \begin{bmatrix}
0 & 0 \\
-f_n(t) \frac{\pi^2 n^2 I_{mn}}{L^4} & 0
\end{bmatrix}
\]

Material properties were assumed using a 126-meter by 31-meter aluminum barge as a model. With a Young’s modulus of \( E_o = 6.9 \times 10^9 \) N/m\(^2\) and \( \mu_o = 1.612 \) kg/m, the true value of the stiffness material property was set equal to

\[
\theta_1 = \frac{E_o I_o}{\mu_o} = 1.3822 \times 10^{12} \frac{m^4}{s^2}
\]

Initial estimates for the model were defined as

\[
B = [1, 0]^T; C = [1, 0]; Q = R = 0.01
\]

In order to validate the algorithm’s ability to discern damage in the material parameters, the control vector \( \bar{u} \) was defined to drive the system towards \( 1.2\theta_1 \). The observation vector \( \bar{y} \) calculated the true value of \( \theta_1 \) for the time series. The Extended Kalman Filter algorithm would be validated if the \( \bar{y} \) measurements identified the over-estimation present from the control signal and then corrected to match the true stiffness material parameter. The resulting estimation for one mode is shown in Figure 2-6 and because the material parameters have been defined to not vary with mode,
this plot is the same as the modes corresponding to \( N = 2 : 5 \).

Figure 2-6: Change in Beam Parameter \( \theta_1 \) at Mode \( N = 1 \)
As expected, with a control input of $1.2\theta_1$ the response does not initially match, but shortly converges to the true value of the beam parameter. In order to validate detection of further changes in the time domain, the true value of the stiffness material parameter was decreased to one-half of the initial value to $\theta_1 = 6.911 \times 10^{11} \frac{m^4}{s^2}$ halfway through the time domain. The results of this change are shown for one mode in Figure 2-7. Similar to Figure 2-6, these results are also representative of modes $N = 2:5$ and demonstrate the algorithm’s ability to correctly detect the true material parameter.

Figure 2-7: Change in Beam Parameter $\theta_1$ at Mode N = 1
2.2 Inhomogeneous Problem Formulation

After considering the beam with homogeneous material properties, the model was developed to address inhomogeneity in material properties and the effect these properties would have on damage detection within the Extended Kalman Filter. An inhomogeneous beam with length $L$, varying moment of inertia, $I(x)$, varying modulus of elasticity $E(x)$, and constant material density, $\mu$ was introduced. The following terms are used to describe different beam parameters in this model:

- a) Modal forcing - The force used to excite the beam, $q_n(t)$. In the Extended Kalman Filter, the control vector $\tilde{u}$ includes this excitative force.

- b) Modal response - The behavior of the beam due to a spatially dependent excitation at a mode, $N$. (For purposes of this investigation, the modal responses are cumulative, i.e. Mode 2 includes the response from Mode 1.)

- c) Modal coefficient - The temporally-dependent portion of the modal response, $C_nf_n(t)$, that modifies the spatial function $g_n(x)$.

2.2.1 Governing ODE

Similar to the homogeneous case, the objective of the inhomogeneous model remains the identification of areas of damage in the beam by detecting significant change in the structural parameter $E(x)I(x)/\mu_o$. The governing equation for an Euler-Bernoulli beam, Equation 2.1, was modified to reflect variations in the modulus of elasticity and second moment of area and a constant material density. The resulting equation...
was
\[
\mu_o \left( \frac{d^2 w(t, x)}{dt^2} \right) + c \left( \frac{dw(t, x)}{dt} \right) + E(x) I(x) \left( \frac{d^4 w(t, x)}{dx^4} \right) = q(t, x) = \sum_{n=1}^{\infty} q_n(t) g_n(x) \quad (2.3)
\]

where \( \mu_o \) is the mass per unit length,

\( c \) is the damping coefficient

\( E(x) \) is the elastic modulus at location \( x \)

\( I(x) \) is the second moment of area at location \( x \)

\( q(t, x) \) is the transverse excitation at time \( t \) and location \( x \)

\( g_n(x) \) is the non-trivial spatial solution \( \sin(\frac{\pi nx}{L}) \).

Simplification of this governing equation followed the same process as introduced with the homogeneous beam including the application of simply-supported boundary conditions, separation of spatial and temporal functions, and the introduction of an orthogonality term. As a result, the governing equation in the case of the inhomogeneous beam in the event of free vibration simplified to

\[
\sum_{n=1}^{N} \left[ f''_n(t) \left( \frac{\mu_o}{2} \right) + f'_n(t) \left( \frac{c}{2} \right) + \frac{1}{2} \left( \frac{\pi^4 n^4}{L^4} \right) E(x) I(x) \right] = 0
\]

where \( N \) is the number of modes that will be kept.

\subsection*{2.2.2 Extended Kalman Filter Formulation: Inhomogeneous Case}

Extended Kalman Filter for Modal Response

Using the same method as the homogeneous case, a first-order model for modal response was converted to a state-space form.

\[
\begin{align*}
F_n'(t) &= A \ddot{x} + B (\ddot{u} + \ddot{w}) \\
y &= C \ddot{x} + \ddot{v}
\end{align*}
\]
This linear system in state-space form was also based on the solution for acceleration subjected to free vibration. The acceleration term in the inhomogeneous case was solved for as

\[ f''_n(t) = -f'_n(t) \frac{c}{\mu_o} - f_n(t) \left[ \frac{E(x)I(x)}{\mu_o} \left( \frac{\pi^4 n^4}{L^4} \right) \right] \]

where beam deflection and velocity for \( n = N \) desired modes was defined as \( F_n(t) = [f_n(t), f'_n(t)]^T \). By setting \( f'_n(t) = h_n \), the first-order model was written as

\[
\begin{align*}
& f'_n(t) = h_n(t) \\
& h'_n(t) = -h_n(t) \frac{c}{\mu_o} - f_n(t) \frac{E(x)I(x)}{\mu_o} \left( \frac{\pi^4 n^4}{L^4} \right)
\end{align*}
\]

The only significant changes made to this state-space form were in the state dynamics matrix \( A \), which was represented as

\[
A = \begin{bmatrix}
0 & I_{mn} \\
-\frac{E(x)I(x)}{\mu_o} \frac{\pi^4 n^4}{L^4} I_{mn} & -\frac{c}{\mu_o} I_{mn}
\end{bmatrix}
\]

where each modal iteration included all previous modes. The other change to this system of equations was to define the control vector \( \tilde{u} \) by the modal forcing

\[
\tilde{u} = \begin{bmatrix} 0 \\ q_n(t) \end{bmatrix}
\]

where \( q_n(t) = \sin(t/5) + 0.3\sin(t/10) + 0.2\sin(t/3) + 0.5\sin(t/7) \). Gaussian noise additions (\( \tilde{w} \) and \( \tilde{v} \)) and was assumed to closely approximate the acceleration of the beam (assumed to be sinusoidal).

The Extended Kalman Filter approach was used to identify the \( F(t) \) states where \( N = 3 \) modes for a time scale of approximately 120 seconds. The initial estimates for the state and covariance were assumed to be

\[
\hat{x} = [0; 0]
\]

\[
P = BQB' = [0; 1]Q[0, 1]
\]
Measurement sensitivity $C$, measurement noise $Q$, and process noise $R$, were assumed to be

$$ C = [1; 0] $$

$$ Q = R = 0.01 $$

The Extended Kalman Filter algorithm introduced in Section 2.1.2 was followed. The resulting estimations for three modal coefficients of the beam are shown in Figures 2-9 to 2-11.

![Modal Coefficient, Mode n = 1](image1)

![Error](image2)

Figure 2-9: Modal Coefficient for $n = 1$, Measuring Mode $N = 1$ Modes
Figure 2-10: Modal Coefficient for $n = 2$, Measuring Mode $N = 2$ Modes

Figure 2-11: Modal Coefficient for $n = 3$, Measuring Mode $N = 3$ Modes
Extended Kalman Filter for Material Parameter Response

Having solved for the modal coefficients, the Extended Kalman Filter was used again to solve for variations in the beam material parameters according to the same procedure introduced in Section 2.1.2. Setting \( \frac{E(x)I(x)}{\mu_0} = r \), the first-order model was

\[
\begin{aligned}
    r'_n &= 0 \\
    h'_n(t) &= -h_n(t)\frac{c}{\mu_0} - f_n(t)r_n\left(\frac{\pi^4n^4}{L^4}\right)
\end{aligned}
\]

No significant changes were made to the model’s state dynamics, stiffness material property, nor initial estimates. However, as a test to validate the model’s ability to discern damage, the control input \( \tilde{u} \) was defined to drive the system towards \( 1.25\theta \). The observation vector \( \tilde{y} \) calculated the true value of \( \theta \). The results presented in Figures 2-12 to 2-14 are presented in the spatial domain vice the time domain. This representation was done as both an a priori reference measurement and a validation that multiple areas of damage were identified. The resulting estimations between subsequent modes do not change as the stiffness material parameter was not defined to vary with mode.
Figure 2-13: Beam Parameter $\theta$ with $N = 2$ Modes Measured

Figure 2-14: Beam Parameter $\theta$ with $N = 3$ Modes Measured
Inhomogeneous material properties were represented in the time-space domain with Modes 1 through 3 shown in Figures 2-15 through 2-17. Each mode captures the two areas of damage that were introduced in the spatial domain as well as a 50% reduction in $\theta_n$ in the time domain. The real-life analogue would be an event of significant damage, possibly a soft ship grounding. These surfaces helped visually validate the responses at various modes.

Figure 2-15: Beam Parameter $\theta$ with $N = 1$ Modes Measured in Space-Time Domains
Figure 2-16: Beam Parameter $\theta$ with $N = 2$ Modes Measured in Space-Time Domains

Figure 2-17: Beam Parameter $\theta$ with $N = 3$ Modes Measured in Space-Time Domains
2.3 Extended Kalman Filter Model Limitations

The Extended Kalman Filter model presented in this chapter show that the Kalman Filter methodology is an appropriate choice for structural damage identification. However, the model is limited in its ability to discern the effect of coupling between modes with an increase in \( N \). The results presented show the cumulative effect of these modes which does not help an end-user (the naval architect or ship maintainer) understand the frequencies that may cause damage. With too much forensic work required to understand the real-life analogue, ultimately, the usefulness of this model is limited. While it is possible to understand the natural frequencies of the perfect beam, the damaged beam requires further analysis. This analysis will be accomplished with the Ensemble Kalman Filter (EnKF) methodology that is well-suited for highly nonlinear systems. Transition to this model is discussed in Chapter 3.
Chapter 3

Nonlinear Damage Identification

3.1 Problem Formulation

Due to limitations in the Extended Kalman Filter methodology, a new method for structural damage identification was formulated with the Ensemble Kalman Filter (EnKF). Like the Extended Kalman Filter, the foundational work of using an Euler-Bernoulli beam with inhomogeneous material properties was maintained. However, this model was updated using data provided by Naval Surface Warfare Center Carderock Division (NSWC CD) for the High Speed Vessel (HSV 2) Swift [3]. The HSV-2 SWIFT was a hybrid catamaran built from type 6082 aluminum. A summary of her principal dimensions and material properties are shown in Table 3.1. Derived characteristics include the second moment of area \( I \approx 15.1 \text{ m}^4 \) and linear density \( \mu \). These values were used in the solution to the governing differential equation and resulting EnKF model for the determination of damping and stiffness.
Table 3.1: HSV-2 SWIFT Characteristics

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length of Water Line (LWL)</td>
<td>92 m</td>
</tr>
<tr>
<td>Length of Frame</td>
<td>3.94 m</td>
</tr>
<tr>
<td>Beam of Side Hull (B)</td>
<td>4.5 m</td>
</tr>
<tr>
<td>Draft (D)</td>
<td>3.43 m</td>
</tr>
<tr>
<td>Young’s Modulus (E)</td>
<td>71 GPa</td>
</tr>
<tr>
<td>Density ($\rho$)</td>
<td>2710 kg/m$^3$</td>
</tr>
</tbody>
</table>

3.1.1 Governing ODE

The governing equation for the response of an Euler-Bernoulli beam to modal forcing may be defined as an alternative form of Equation 2.3

\[
\begin{align*}
\frac{d^2q_n(t)}{dt^2} &+ c f_n(t) \int_0^L \mu(x) g_m(x) g_n(x) dx + f_n(t) \int_0^L E(x) I(x) \frac{d^2g_n(x)}{dx^2} dx = q_n(t) \\
q_n(t) &= \int_0^L q(t, x) g_n(x) dx
\end{align*}
\]

where $f_n(t)$ is representative of the time derivative

- $\mu(x)$ is the mass per unit length at location $x$
- $g_m(x)$ and $g_n(x)$ are the non-trivial spatial solutions $\sin\left(\frac{\pi m}{L} x\right)$.
- $c$ is the damping coefficient
- $E(x)$ is the elastic modulus at location $x$
- $I(x)$ is the second moment of area at location $x$
- $q(t, x)$ is the transverse excitation at time $t$ and location $x$

To accurately account for forcing in the spatial domain, forcing ($q(t, x)$) was defined as

\[
q(t, x) = q_n(x) \sin(\tilde{\omega} t)
\]

The time-dependency of the forcing variable was introduced by using a vector of excitation frequencies split into two frequency domains, $\tilde{\omega} = [1, 5]$. The two corresponding
spatial components were defined as predetermined, but arbitrary vectors. The spatial components, \( q_n(x) \) were defined as

\[
q_1(x) = 0.8\sin\left(\frac{\pi x}{L}\right) + 0.7\sin\left(\frac{3\pi x}{L}\right) + 0.8\sin\left(\frac{5\pi x}{L}\right) + 0.8\sin\left(\frac{7\pi x}{L}\right) + 0.8\sin\left(\frac{9\pi x}{L}\right)
\]

\[
q_2(x) = 0.6\sin\left(\frac{2\pi x}{L}\right) + 0.8\sin\left(\frac{4\pi x}{L}\right) + 0.6\sin\left(\frac{6\pi x}{L}\right) + 0.7\sin\left(\frac{8\pi x}{L}\right) + 0.6\sin\left(\frac{10\pi x}{L}\right)
\]

The resultant modal forcing was defined as

\[
Q_n(t) = \sin(\omega t) \int_0^L \begin{bmatrix} q_1(x) & q_2(x) \end{bmatrix} \sin\left(\frac{n\pi x}{L}\right) dx
\]

To further simplify the governing equation, the following substitutions were made

\[
A_{mn} = \int_0^L \mu(x)\sin\left(\frac{\pi mx}{L}\right)\sin\left(\frac{\pi nx}{L}\right) dx
\]

\[
B_{mn} = c \int_0^L \sin\left(\frac{\pi mx}{L}\right)\sin\left(\frac{\pi nx}{L}\right) dx
\]

\[
C_{mn} = \int_0^L E(x)I(x)\sin\left(\frac{\pi mx}{L}\right)\left(\frac{\pi^4 n^4}{L^4}\right)\sin\left(\frac{\pi nx}{L}\right) dx
\]

Final substitution into the governing equation yields

\[
A_{mn}f''_n(t) + B_{mn}f'_n(t) + C_{mn}f_n(t) = Q_n(t)
\]

To utilize MATLAB’s ode45 function, the following system of equations was set up to solve for \( f_n(t) \)

\[
\begin{cases}
  f_1(t) = f(t) \\
  f_2(t) = f'(t)
\end{cases}
\]

\[
\begin{cases}
  f_1'(t) = f_2(t) \\
  f_2'(t) = \frac{1}{A_{mn}}(-B_{mn}f_2'(t) - C_{mn}f_1(t) + Q_n(t))
\end{cases}
\]

Initially, damping of the beam \( (c) \) was assumed to be zero and the distributed values \( (E(x), I(x), \text{ and } \mu(x)) \) were predetermined in order to introduce damage into
the model. Within the EnKF model, damage was simulated by changing the term $E(x)I(x)$ to 80% of the original value from $L = 60$ meters to $L = 80$ meters. At all other locations this term was left at the original value ($10.7 \times 10^{11} \text{ N} - \text{m}^2$). The desired number of modes was set at $N_d = 5$ with $n = 5$ and $m = 5$ for a beam of length, $L = 92$ meters and time of $t = 100$ seconds. Higher modes ($n \geq 5$) were not considered in the initial formulation of the damaged model. Initial conditions were set as: $f_n(t) = 0$ and $f'_n(t) = 0$. $Q_n(t)$ was evaluated with time dependency from 1 to 100 seconds and Gaussian noise was not initially added to $Q_n(t)$. After a solution was determined, a plot of the modal responses, $f_n(t)$, were compared in Figure 3-1.

Figure 3-1: Modal Response for $n = 5$, Measuring $N = 5$ Modes
3.2 Ensemble Kalman Filter Formulation

The set-up of the EnKF began with the identification of the desired state, $F$, to include displacement $f_n(t)$, velocity $f'_n(t)$, damping $c_n$, stiffness $k_n$, as well as the coupling modes $C_{mn}$. To achieve a simultaneous solution, the state was required to include all modal information as well as all coupling modes. This state was defined as

$$F = \begin{bmatrix} f_1 & f'_1 & c_1 & k_1 & \ldots & f_n & f'_n & c_n & k_n & C_{11} & C_{12} & \ldots & C_{mn} \end{bmatrix}^T$$

Other initial definitions included the state covariance matrix ($N_S$), the observability matrix ($H$) where position was the only input, and the desired number of ensemble members which was set to $N = 200$. For $n = 2$ desired modes, $N_S$ and $H$ were defined as: $N_{S2} = $ 

$$H_2 = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T$$

In the EnKF, the true state is approximated by using the ensemble members. The prior ensemble is defined as a $[m \times N]$ matrix where $m$ is the number of states ($N_d(N_d + 4)$ in this case) and $N$ is the number of ensemble members (200). For the length of an observation vector ($\tilde{z}$), defined to be the solution to the modal forcing ODE, the posterior ensemble and state are updated in the following manner:

1. The prior ensemble is modified to include a time-influenced noise factor ($N_X$),
comprised of parameter noise, \( N_{PAR} \), and plant dynamics noise, \( N_{EQS} \).

\[
F = \begin{bmatrix}
F_1^u \\
F_2^u \\
F_3^u \\
F_4^u \\
F_{4n+1:4n+n^2}^u
\end{bmatrix} + \sqrt{dt}N_X
\]

where \( F_m^u \) is composed of the updated ensemble members as discussed in Section 3.2.1.

2. Next, the posterior ensemble is calculated including a measurement data vector \( (D) \) that has had observation noise \( (N_{OBS}) \) added.

\[
F_{(k|k)} = F_{(k|k-1)} + \left[ P_{(k|k-1)}H^T(HP_{(k|k-1)}^TH^T + N_{OBS})^{-1}\right] \left[ D - HF_{(k|k-1)} \right]
\]

3. The current state, \( F_{k|k} \), is defined as the mean of the ensemble members, mean\((F)\), and the current covariance, \( P_{k|k} \), is defined as the covariance of the ensemble members \( cov(F) \).

### 3.2.1 Ensemble Member Update

Within the Ensemble Kalman Filter, a series of functions were set up to account for updates to the displacement and velocity ensemble members. The original ensemble state is modified by the states of other ensemble members and a control vector \( (\vec{u}) \). The control vector is an indication of the given forcing and has been set to zero due to its relatively slow response.

\[
\begin{bmatrix}
f_n \\
f_n' \\
c_n \\
k_n \\
C_{nn}
\end{bmatrix} = \begin{bmatrix}
F_{1n}^u \\
F_{2n}^u \\
F_{3n}^u \\
F_{4n}^u \\
F_{4n+1:4n+n^2}^u
\end{bmatrix} = \begin{bmatrix}
F_{1n} + (dt)F_2 \\
F_{2n} + dt(-F_3F_2 - F_4F_1 + \vec{u}) \\
F_3n \\
F_4n \\
F_{4n+1:4n+n^2}
\end{bmatrix}
\]
3.2.2 Noise Parameter Optimization

The choice of noise parameters within the algorithm is a critical step for the proper propagation of state and covariance within the EnKF. The parameter noise must be chosen to be comparable to the forcing for each mode while recognizing that the parameter noise will have an inverse impact on the weighting of observations and dynamical plant (system) noise. In addition to these noise parameter choices, an appropriate time step must be chosen to have properly sampled data at all modes. The results seen in Figures 3-2 to 3-6 have been built using a time step of $dt = 0.001$ seconds, due to the fast response of the system (as shown in Figure 3-1). In an attempt to optimize the EnKF’s estimation with actual measurements for all modes, the noise parameters were changed to vary with each mode. Values for each mode are shown in Table 3.2.

Table 3.2: Noise Parameters

<table>
<thead>
<tr>
<th>Mode</th>
<th>$N_{Eqs}$</th>
<th>$N_{Par}$</th>
<th>$N_{OBS}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mode 1</td>
<td>0.01</td>
<td>$\frac{1}{4} A_n$</td>
<td>0.05</td>
</tr>
<tr>
<td>Mode 2</td>
<td>0.001</td>
<td>$\frac{1}{4} A_n$</td>
<td>0.005</td>
</tr>
<tr>
<td>Mode 3</td>
<td>0.001</td>
<td>$\frac{1}{4} A_n$</td>
<td>0.005</td>
</tr>
<tr>
<td>Mode 4</td>
<td>0.001</td>
<td>$\frac{1}{4} A_n$</td>
<td>0.005</td>
</tr>
<tr>
<td>Mode 5</td>
<td>0.001</td>
<td>$\frac{1}{4} A_n$</td>
<td>0.005</td>
</tr>
</tbody>
</table>
3.2.3 Ensemble Kalman Filter Results

The results of the Ensemble Kalman Filter for damping and stiffness are shown in Figures 3-2 and 3-3. As can be seen, this formulation is not optimal for identification of the damaged beam’s material properties.

Figure 3-2: Material Response for n = 1, Measuring N = 2 Modes

Figure 3-3: Material Response for n = 2, Measuring N = 2 Modes
Attempts to include higher modes, radically impact the results seen in lower modes. For a simultaneous run of 3 desired modes, the following results are generated:

Figure 3-4: Material Response for $n = 1$, Measuring $N = 3$ Modes

Figure 3-5: Material Response for $n = 2$, Measuring $N = 3$ Modes
Figure 3-6: Material Response for n = 3, Measuring N = 3 Modes

3.3 Ensemble Kalman Filter Model Limitations

As shown in Figures 3-2 to 3-6, only some of the damaged area is captured in the results if the value of desired modes (N) is set to a low value. While the EnKF captures order-one displacements the best, it should be noted that forcing across the length of a ship does not result in order-one modal displacements. Re-tuning of noise parameters was able to capture some of the state data from the EnKF (Figures 3-2 and 3-3). However, this was a time-consuming process done through individual optimization of each mode’s noise parameters. It would not be feasible in the field without a time-intensive setup and an initial validation data set. However, based off of known naval architecture practices and material maintenance conditions, there are most likely areas for damage onboard a ship. The EnKF methodology shows that it is possible to capture structural damage, but is difficult to achieve fine-tuned resolution between material parameters. There is a need to couple this EnKF identification methodology with pre-existing naval design knowledge to fully capture the parametric non-linearities present with the damaged beam model.
Chapter 4

Scenario-Driven Damage Identification

4.1 Problem Formulation

To try and fully capture the extent of damage onboard a ship, it is hypothesized that the most likely areas of damage will be informed by a previously known damage scenario. This previously known damage scenario can be defined by a review of classifications societies’ (such as American Bureau of Shipping) standards for Owner’s Hull Inspections and Maintenance Schemes. Within the US Navy, certain ship classes may expect to see similar structural damage. As an example, onboard the US Navy’s CG-47 Ticonderoga Class Cruisers, hull damage was seen in the forward sections of the ship from slamming as well as corrosion fatigue along aluminum superstructure areas ([5],[25]). Images of these damage areas are shown in Figures 4-1 and 4-2.
Figure 4-1: Forward Structural Damage on CG-47 Class, [5]

Figure 4-2: Superstructure Structural Damage on CG-47 Class, [25]
To develop a scenario-driven damage identification scheme, it is hypothesized that the intensity of damage will be controlled within the structure. This will be represented by

\[ \tilde{C} = \tilde{C} + y_1\tilde{C}_1 + y_2\tilde{C}_2 + \ldots + y_n\tilde{C}_n \]

where \( \tilde{C}_n \) is the undamaged stiffness matrix

\[ \tilde{C} \] is the damaged stiffness matrix at a position, \( x \)

\( y_n \) is a damage factor measured in percent reduction from the original stiffness matrix

The position, \( x \), where damage can be expected to occur onboard a naval vessel is informed from some general naval architecture guidelines. Building off the discussion of sea loading presented in Section 1.1, likely damage areas are assumed to occur proportionally to areas that experience high sea loading effects. For purposes of this research, damage is assumed to occur first in areas of high-frequency dynamic and impact loading between main transverse bulkheads with particular emphasis on the bow from slamming and machinery room areas. Using this conjecture, the EnKF model has been tailored to try and identify damage with specific scenarios.

Using the refined EnKF model, scenarios of damage are implemented in single areas of the hull and then expanded to include multiple areas of damage. For a scenario with a single area of damage, a damaged area (30% reduction in original stiffness) was input between 60 meters to 80 meters (0.65L to 0.87L) of the model defined in Section 3.1 and results are discussed in Section 4.2. For a scenario with multiple areas of damage, three sections of potential damage were input between 9.2 meters and 18.5 meters (0.1L to 0.2L), 27.6 meters and 41.5 meters (0.3L to 0.45L), and 55.4 meters to 73.8 meters (0.6L to 0.8L). The true state of damage included a 30% reduction in the first location, a 20% reduction in the second location, and no damage in the third location. Results of this scenario are discussed in Section 4.3.
### 4.1.1 Governing ODE

As shown in Section 3.1.1, the governing equation for the response of an Euler-Bernoulli beam to modal forcing is defined as

\[
f''_n(t) \int_0^L \mu(x)g_m(x)g_n(x)dx + cf'_n(t) \int_0^L g_m(x)g_n(x)dx + f_n(t) \int_0^L E(x)I(x)g_m(x)\frac{dg_n(x)}{dx}dx = q_n(t)
\]

where \( q_n(t) = \int_0^L q(x,t)g_n(x)dx \)

where \( f'_n(t) \) is representative of the time derivative

\( \mu(x) \) is the mass per unit length at location \( x \)

\( g_m(x) \) and \( g_n(x) \) are the non-trivial spatial solutions \( \sin\left(\frac{\pi(m,n)x}{L}\right) \).

\( c \) is the damping coefficient

\( E(x) \) is the elastic modulus at location \( x \)

\( I(x) \) is the second moment of area at location \( x \)

\( q(t, x) \) is the transverse excitation at time \( t \) and location \( x \)

Following the EnKF derivations, the governing equation was simplified to

\[
A_{mn}f''_n(t) + B_{mn}f'_n(t) + C_{mn}f_n(t) = Q_n(t)
\]

where

\( A_{mn} = \int_0^L \mu(x)\sin\left(\frac{\pi mx}{L}\right)\sin\left(\frac{\pi nx}{L}\right)dx \)

\( B_{mn} = c \int_0^L \sin\left(\frac{\pi mx}{L}\right)\sin\left(\frac{\pi nx}{L}\right)dx \)

\( C_{mn} = \int_0^L E(x)I(x)\sin\left(\frac{\pi mx}{L}\right)\left(\frac{\pi n^4}{L^4}\right)\sin\left(\frac{\pi nx}{L}\right)dx \)

\( Q_n(t) = \sin(\tilde{\omega}t) \int_0^L \left[ q_1(x) q_2(x) \right] \sin\left(\frac{\pi nx}{L}\right)dx \)

\( Q_n(t) \) remains the same definition as in Section 3.1.1.

Within the scenario-driven identification scheme, this equation is modified to

\[
A_{mn}f''_n(t) + B_{mn}f'_n(t) + (\tilde{C}_0 + y_1\tilde{C}_1)f_n(t) = Q_n(t)
\]
To solve for $f_n(t)$, the following system of equations was set up to utilize MATLAB’s ode45 function

\[
\begin{aligned}
&f_1(t) = f(t) \\
f_2(t) = f'(t) \\
f'_1(t) = f_2(t) \\
f'_2(t) = \frac{1}{A_{mn}} (-B_{mn} f'_2(t) - C_{mn} f_1(t) + Q_n(t))
\end{aligned}
\]

4.1.2 Higher Modes Resolution: Rescaling

On a first pass of the above formulation, the parameter selection was insufficient for convergence. The stiffness matrix, $C_{mn}$, originally of the order of $10^{10}$ caused divergence within the EnKF. Rescaling was necessary to allow for convergence of the Ensemble Kalman Filter; this was done according to the following principle

\[
\tau = \alpha t
\]

\[
\alpha^2 x''_1 + \omega^2_{0,1} x_1 = 0
\]

\[
x''_1 + x_1 = 0
\]

\[
x''_2 + \frac{\omega^2_{0,2}}{\omega^2_{0,1}} x_2 = 0
\]

As a result, the governing ODE was defined as

\[
f''_n(t) + \frac{B_{mn}}{A_{mn}} f'_n(t) + \frac{C_{mn}}{A_{mn}} f_n(t) = \frac{Q_n(t)}{A_{mn}(t)}
\]

Due to the lack of damping within the model, the second term was dropped resulting in

\[
f''_n(t) + \frac{C_{mn}}{A_{mn}} f_n(t) = \frac{Q_n(t)}{A_{mn}(t)}
\]

\[
f''_n(t) + \frac{\pi^4 n^4 E(x) I(x)}{L^4 \mu(x)} f_n(t) = \frac{Q_n(t)}{A_{mn}(t)}
\]
The variable of interest is the variable modifying the \( f_n(t) \) which defines the fundamental frequency of the system. Setting

\[
\omega_{0,1}^2 = \alpha = \sqrt{\frac{\pi^4 E(x) I(x)}{L^4 \mu(x)}}
\]

the following equations were modified

\[
\tilde{\omega} = [1, 5]/\alpha
\]

\[
Q_n(t) = \sin(\omega t) \int_0^L \left[ \frac{q_1(x)}{\alpha^2} - \frac{q_2(x)}{\alpha^2} \right] \sin\left(\frac{n\pi x}{L}\right) dx
\]

\[
A_{mn}f''_n(t) + B_{mn}f'_n(t) + \left( \frac{\tilde{C}_o}{\alpha^2} + y_1 \frac{\tilde{C}_1}{\alpha^2} \right) f_n(t) = Q_n(t)
\]

As a result of this rescaling, the stiffness matrices \( \tilde{C}_o \) and \( \tilde{C}_1 \) were reduced to order one and the system of equations defining the original ODE were used to solve for the modal deflection. The desired number of modes for an analysis of a single point of damage was set at \( N_d = 3 \) with \( n = 3 \) and \( m = 3 \) for a beam of length, \( L = 92 \) meters and time of \( t = 100 \) seconds. Initial conditions were set as: \( f_n(t) = 0 \) and \( f'_n(t) = 0 \). \( Q_n(t) \) was evaluated with time dependency through 1 to 100 seconds and Gaussian noise was not initially added to \( Q_n(t) \). After a solution was determined, a plot of the modal responses, \( f_n(t) \), were compared in Figure 4-3.
Figure 4-3: Modal Response for $n = 3$, Measuring $N = 3$ Modes
4.2 Ensemble Kalman Filter: Single Area of Damage

The Ensemble Kalman Filter was designed to determine displacement, velocity, and the damage factor measured in percent reduction. The set-up of the Ensemble Kalman Filter began with the identification of the desired state, $F$, to include displacement ($f_n(t)$), velocity ($f'_n(t)$), and damage ($y_n$). The state was defined as

$$F = \begin{bmatrix} f_1 & f_2 & \ldots & f_n & f'_1 & f'_2 & \ldots & f'_n & y_1 \end{bmatrix}^T$$

Other initial definitions included the state covariance matrix ($N_S$), the observability matrix ($H$) where position was the only input, and the desired number of ensemble members which was set to $N = 2000$. For $N_d = 2$, $N_S$ and $H$ were defined as

$$N_S = \begin{bmatrix} N_{EQS_1}^2 & 0 & 0 & 0 & 0 \\ 0 & N_{EQS_2}^2 & 0 & 0 & 0 \\ 0 & 0 & N_{EQS_1}^2 & 0 & 0 \\ 0 & 0 & 0 & N_{EQS_2}^2 & 0 \\ 0 & 0 & 0 & 0 & N_{PAR_1}^2 \end{bmatrix}$$

(4.1)

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

Within the Ensemble Kalman Filter, the true state is approximated by using the ensemble members. The prior ensemble is defined as a $[m \times N]$ matrix where $m$ is the number of states ($2N_d + 1$ in this case) and $N$ is the number of ensemble members (2000). For the length of an observation vector ($\tilde{z}$), defined to be the solution to the modal forcing ODE, the posterior ensemble and state are updated in the following manner.

1. The prior ensemble is modified to include a time-influenced noise factor ($N_X$), comprised of a mean vector of zeroes and the state covariance matrix defined in
Equation 4.1 consisting of parameter noise, $N_{PAR}$, and plant dynamics noise, $N_{EQS}$.

\[
F = \begin{bmatrix} F_1^u \\ F_2^u \\ F_3^u \end{bmatrix} + \sqrt{dt}N_X
\]

where $F_m^u$ is composed of the updated ensemble members as discussed in Section 4.2.1.

2. Next, the posterior ensemble is calculated including a measurement data vector $(D)$ that has had observation noise $(N_{OBS})$ added.

\[
F_{(k|k)} = F_{(k|k-1)} + \left[ P_{(k|k-1)}HT(HP_{(k|k-1)}^TH^T + N_{OBS}^2)^{-1} \right] [ D - HF_{(k|k-1)} ]
\]

3. The current state, $F_{k|k}$, is defined as the mean of the ensemble members, $mean(F)$ and the current covariance, $P_{k|k}$ is defined as the covariance of the ensemble members $cov(F)$.

### 4.2.1 Ensemble Member Update

Within the Ensemble Kalman Filter, a series of functions were set up to account for updates to the displacement and velocity ensemble members. The original ensemble state was modified using the coefficients from the ODE, $A_n$, $B_n$, and $Q_n(t)$. $C_o$ is the undamaged stiffness matrix and $C_{mn}$ is the damaged stiffness matrix.

\[
\begin{bmatrix}
    f_n \\
    f'_n \\
    y_1
\end{bmatrix} = \begin{bmatrix} F_1^u \\ F_2^u \\ F_3^u \end{bmatrix} = \begin{bmatrix}
    F_1 + dtF_2 \\
    F_2 + \frac{dt}{A_n}(-B_nF_2 - (C_o + F_3C_{mn})F_1 + Q_n(t)) \\
    F_3
\end{bmatrix}
\]

As an example of the above code, for $N_d = 2$ modes, the implementation where
\( \bar{C} \) is the undamaged matrix and \( F_n \) is the ensemble representation is

\[
\begin{bmatrix}
  f_1 \\
  f_2 \\
  f'_1 \\
  f'_2 \\
  y_1
\end{bmatrix} =
\begin{bmatrix}
  F_1 + dtF_3 \\
  F_2 + dtF_4 \\
  F_3 + dt(-\bar{C}_{11}F_1 - F_5C_{11}F_1 - F_5C_{12}F_2 + Q_1(t)) \\
  F_4 + dt(-\bar{C}_{22}F_2 - F_5C_{21}F_1 - F_5C_{22}F_2 + Q_2(t)) \\
  F_5
\end{bmatrix}
\]

4.2.2 Noise Parameter Optimization

As discussed in Section 3.2.2, the choice of noise parameters within the algorithm is a critical step for the proper propagation of state and covariance within the Ensemble Kalman Filter. The observation noise is chosen to be approximately ten times smaller than the mode’s fluctuations (shown in Figure 4-3). The parameter noise is chosen to be approximately 1% of the actual damage value. In addition to the noise parameter choices, an appropriate time step must be chosen to have properly sampled data at all modes. The results seen in Figures 4-4 to 4-5 have been built using a time step of \( dt = 0.01 \) seconds, due to the fast response of the system (as seen in Figure 4-3). In an attempt to optimize the EnKF’s estimation with actual measurements for all modes, the noise parameters were changed to vary with each mode. Values for each mode are shown below in Table 4.1.

<table>
<thead>
<tr>
<th>Mode</th>
<th>( N_{EQS} )</th>
<th>( N_{PAR} )</th>
<th>( N_{OBS} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mode 1</td>
<td>( 1 \times 10^{-4} )</td>
<td>( 1 \times 10^{-2} )</td>
<td>( 5 \times 10^{-3} )</td>
</tr>
<tr>
<td>Mode 2</td>
<td>( 1 \times 10^{-4} )</td>
<td>( 1 \times 10^{-2} )</td>
<td>( 1 \times 10^{-3} )</td>
</tr>
<tr>
<td>Mode 3</td>
<td>( 1 \times 10^{-4} )</td>
<td>( 1 \times 10^{-2} )</td>
<td>( 1 \times 10^{-3} )</td>
</tr>
</tbody>
</table>
4.2.3 Ensemble Kalman Filter Results

The results of the Ensemble Kalman Filter for position and damage for both two modes and three modes are shown in Figures 4-4 through 4-7. Notably, there is convergence in both the position and damage estimates of the Ensemble Kalman Filter with the actual measurements.

Figure 4-4: Position Response for n = 2, Measuring N = 2 Modes

Figure 4-5: Damage Response for n = 2, Measuring N = 2 Modes
Figure 4-6: Position Response for $n = 3$, Measuring $N = 3$ Modes

Figure 4-7: Damage Response for $n = 3$, Measuring $N = 3$ Modes
4.3 Ensemble Kalman Filter: Multiple Damage Areas

After validation of the EnKF for a single damage area, the algorithm was extended to multiple areas of damage. The code used to generate these results is included in Appendix A. The same setup as described in Section 4.1.1 was used with the calculation being run for three potential damage areas. Results of the modal deflection for a 200 second time scale are shown in Figure 4-8.

![Modal Response for n = 2, Measuring N = 2 Modes](image)

The EnKF was modified to solve for these damage areas by setting the state as

\[
F = \begin{bmatrix} f_1 & f_2 & \ldots & f_n & f'_1 & f'_2 & \ldots & f'_n & y_1 & y_2 & y_3 \end{bmatrix}^T
\]
Noise parameters were modified as shown in Table 4.2.

Table 4.2: Noise Parameters

<table>
<thead>
<tr>
<th>Mode</th>
<th>$N_{EQS}$</th>
<th>$N_{PAR}$</th>
<th>$N_{OBS}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mode 1</td>
<td>$1 \times 10^{-4}$</td>
<td>$5 \times 10^{-3}$</td>
<td>$5 \times 10^{-3}$</td>
</tr>
<tr>
<td>Mode 2</td>
<td>$1 \times 10^{-4}$</td>
<td>$5 \times 10^{-3}$</td>
<td>$1 \times 10^{-3}$</td>
</tr>
<tr>
<td>Mode 3</td>
<td>$1 \times 10^{-4}$</td>
<td>$5 \times 10^{-3}$</td>
<td>$1 \times 10^{-3}$</td>
</tr>
</tbody>
</table>

Ensemble member update was conducted iteratively for the number of suspected damage areas. No other changes were made to the general formulation of the EnKF described in Section 4.2. The results for the multiple areas of damage are shown in Figures 4-9 and 4-10 for two modes of analysis. True damage was $y_1 = 0$, $y_2 = 0.3$, and $y_3 = 0.2$, and results show convergence with these values. Extension to three modes is shown in Figures 4-11 and 4-12.
Figure 4-10: Damage Response for $n = 2$, Measuring $N = 2$ Modes

Figure 4-11: Position Response for $n = 3$, Measuring $N = 3$ Modes
Figure 4-12: Damage Response for $n = 3$, Measuring $N = 3$ Modes
4.4 Scenario-Driven Damage Identification Resolution

Resolution of the scenario-driven damage identification model is approximately 4%. For ship maintenance, this resolution is quite good for almost all structural elements present onboard. Most mandatory repairs will not be required until \( \geq 25\% \) material wastage or a cumulative reduction is seen to the ship’s overall strength as measured by the hull’s section modulus. These action points will vary according to specific ship licensure as governed by respective classification societies. However, within the identification scheme presented in Chapter 4, the resolution provides the ship maintainer with the ability to track damage onboard prior to major incident or overhaul.

The obvious limitation to any scenario-driven identification scheme are the scenarios that were not considered. Within the Ensemble Kalman Filter methodology, displacement, velocity, and a percentage of damage are estimated and identified at certain locations chosen according to known loading conditions and impacts. The chance that a ship may experience structural damage and/or failure in an unusual and previously unknown way is likely to be associated with the “newness” of the ship’s design. That is, ship designs that have proven performance at sea will have more known damage characteristics based off available parametric data than a ship that has not been launched. The “newness” of design may impact the building shipyards’ construction of a particular class of vessel with material impact present from initial welding defects and initial build execution. In such instances, it would be wise to complement identification schemes with traditional methods of visual and ultrasound inspections.

To truly assess this informed EnKF methodology, it is necessary to validate this model against a trial data set. Chapter 5 will focus on the finite element model that was used in the MAESTRO Global Analysis Software to validate this method.
Chapter 5

Validation

5.1 MAESTRO Finite Element Model

Validation of the Scenario Driven Identification-EnKF (SDI-EnKF) model derived in Chapter 4 was conducted using the MAESTRO Global Analysis software suite. This software is a ship structural analysis program that conducts finite element analysis (FEA). The software has the ability to conduct a time-domain analysis of a model vessel through the MAESTRO Wave analysis package. The time-domain analysis results returned computed motion and loading responses which could then be modified to fit the SDI-EnKF model parameters. However, the starting point for the time-domain analysis required a complete model where appropriate loading conditions and “wettable” (i.e. immersed) panels were defined. To meet these requirements, a full-ship sample model of an Offshore Vessel (OSV) was taken from the reference sample library and modified. The model has a length of 50.3 meters, a beam of 11.03 meters, and total height of 11.2776 meters. The vessel can be seen in Figures 5-1 and 5-2.
Figure 5-1: MAESTRO Model Perspective

Figure 5-2: MAESTRO Model Profile
The model was constructed out of isotropic high-strength steel with the properties shown in Table 5.1. The SDI-EnKF distributed properties were updated to reflect these new dimensions and material properties.

Table 5.1: MAESTRO Model Principal Dimensions and Material Properties

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length of Water Line (LWL)</td>
<td>50.292 m</td>
</tr>
<tr>
<td>Beam of Hull (B)</td>
<td>11.03 m</td>
</tr>
<tr>
<td>Height (H)</td>
<td>11.2776 m</td>
</tr>
<tr>
<td>Draft (D)</td>
<td>3.43 m</td>
</tr>
<tr>
<td>Young’s Modulus (E)</td>
<td>204000 ( \frac{\text{MN}}{\text{m}^2} )</td>
</tr>
<tr>
<td>Poisson Ratio</td>
<td>0.3</td>
</tr>
<tr>
<td>Density (( \rho ))</td>
<td>7.85 ( \frac{\text{tonne}}{\text{m}^3} )</td>
</tr>
<tr>
<td>Yield Stress</td>
<td>235 ( \frac{\text{MN}}{\text{m}^2} )</td>
</tr>
<tr>
<td>Ultimate Tensile Strength</td>
<td>400 ( \frac{\text{MN}}{\text{m}^2} )</td>
</tr>
<tr>
<td>Reduced Yield Stress at AL Heat Affected Zone</td>
<td>235 ( \frac{\text{MN}}{\text{m}^2} )</td>
</tr>
<tr>
<td>NVR Allowable Bending Stress</td>
<td>188 ( \frac{\text{MN}}{\text{m}^2} ) (0.8Y)</td>
</tr>
<tr>
<td>HSNC Allowable Yield Stress</td>
<td>235 ( \frac{\text{MN}}{\text{m}^2} )</td>
</tr>
</tbody>
</table>

5.1.1 Time-Domain Analysis Results

Using the baseline model described in Section 5.1, a MAESTRO structural analysis was run for the full finite-element model of 934 elements. The results of this structural analysis provided longitudinal properties as well as the shear force and bending moment for the still-water load condition. A sample representation of the data collected from this process can be seen with the model’s shear force profile in Figure 5-3.

After conducting the structural analysis, the time-domain analysis using MAESTRO Wave was run. For each time-domain analysis, speed, heading, and sea condition were specified. Environmental conditions were defined as a regular wave environment with a one-meter wave height and a ten-second wave period. The vessel’s loading condition was analyzed in a still-water load condition. A nonlinear hydrostatic analysis evaluated velocity potential and Froude-Krylov forces on the instantaneous hull.
wetted surface up to the incident wave. Calculations for motion and load responses of the vessel were conducted for a total time of 850 seconds at 0.25 second intervals. Roll damping could have been accounted for as well within the time-domain analysis, however the SDI-EnKF model assumed zero damping so no damping was accounted for in the model.

The MAESTRO Wave motion and load response calculations included pressure distribution, point forces, accelerations, displacements, velocities, and hull girder loads. The exported data for hull girder loads provided time, location, vertical shear force, and vertical bending moment data. MATLAB was used to generate visuals of this data (Figures 5-4 and 5-5) to understand the initial model.
Figure 5-4: Vertical Shear Force of Undamaged OSV

Figure 5-5: Vertical Bending Moment of Undamaged OSV
5.2 Governing Equations and Formulation

The MAESTRO time-domain analysis returned panel force \( F(x) \) [tonnes], vertical shear force \( VSF(x) \) [tonnes], and vertical bending moment \( VBM(x) \) [tonne-m] for different hull girder loads. Hull properties including the nonlinear observables \( I(x) \) [m\(^4\)] and \( \mu(x) \) [kg/m] were identified and/or derived from MAESTRO. The first step in validating the model was to solve for modal deflection \( w_n(x,t) \) [m] before solving for forcing \( q_n(t) \) [N/m].

Deflection was solved for in accordance with the relationship

\[
VBM(x) = EI(x) \frac{d^2 w(x,t)}{dx^2}
\]

\[
\frac{d^2 w(x,t)}{dx^2} = \frac{VBM(x)}{EI(x)}
\]

\[
w(x,t) = \int_0^x \int_0^y \frac{VBM(y)}{EI(y)} dy dx + C_1(t)x + C_2(t)
\]

The spatial domains \( y \) and \( x \) are both set equal to the vessel’s stations with positive values, i.e. Station 2 at 0.889 m to Station 21 at 48.664 m. The constants were determined from boundary conditions where \( w(x,t) = 0 \) \( |x=0,L \) and \( \frac{d^2 w(x,t)}{dx^2} = 0 \) \( |x=0,L \).

At \( x = 0 \):

\[
w(0, t) = 0 = 0 + C_1(t)(0) + C_2(t)
\]

\[
C_2(t) = 0
\]

At \( x = L \):

\[
w(L, t) = 0 = \int_0^L \int_0^y \frac{VBM(y)}{EI(y)} dy dx + C_1(t)(L)
\]

\[
\int_0^L \int_0^y \frac{VBM(y)}{EI(y)} dy dx = C_1(t)L
\]

\[
C_1(t) = \frac{\int_0^L \int_0^y \frac{VBM(y)}{EI(y)} dy dx}{L}
\]

\[
w(x,t) = \int_0^x \int_0^y \frac{VBM(y)}{EI(y)} dy dx + \frac{\int_0^L \int_0^y \frac{VBM(y)}{EI(y)} dy dx}{L} x
\]
This final equation was implemented in MATLAB by using a Fourier expansion to approximate the double integral

\[ VBM(x) = EI(x) \frac{d^2w}{dx^2} \]

\[ w = \sum A_n \sin \left( \frac{n\pi x}{L} \right) \]

\[ \frac{d^2w}{dx^2} = - \sum A_n \frac{n^2 \pi^2}{L^2} \sin \left( \frac{n\pi x}{L} \right) \]

Substitution back into the governing equation yielded

\[ VBM(x) = EI(x) \sum A_n \frac{n^2 \pi^2}{L^2} \sin \left( \frac{n\pi x}{L} \right) \]

\[ VBM(x) = \sum A_n EI(x) \frac{n^2 \pi^2}{L^2} \sin \left( \frac{n\pi x}{L} \right) \]

To maintain orthogonality, this product was multiplied by \( \sin \left( \frac{m\pi x}{L} \right) \) and integrated. The resulting equation was

\[ \int VBM(x) \sin \left( \frac{m\pi x}{L} \right) dx = \sum A_n \int EI(x) \frac{n^2 \pi^2}{L^2} \sin \left( \frac{n\pi x}{L} \right) \sin \left( \frac{m\pi x}{L} \right) dx \]

Setting

\[ \rho_m = \int VBM(x) \sin \left( \frac{m\pi x}{L} \right) dx \]

\[ S_{mn} = \int EI(x) \frac{n^2 \pi^2}{L^2} \sin \left( \frac{n\pi x}{L} \right) \sin \left( \frac{m\pi x}{L} \right) dx \]

the equation simplifies to

\[ \rho_m = S_{mn} A_n \]

where \( A_n \) is modal deflection. The results for three modes in an undamaged scenario are shown in Figure 5-6.
Using the deflection term, the governing equation for the modal forcing term, \( q_n(t) \) [N/m], was solved for using a similar double series approximation

\[
q(t) = \mu(x) \frac{d^2 w}{dt^2} + \frac{d}{dx^2} \left( EI(x) \frac{d^2 w}{dx^2} \right)
\]

\[
q(t) = -\mu(x) \sum \frac{n^2 \pi^2}{L^2} w_n(t) \sin \left( \frac{n \pi t}{L} \right) + \sum EI(x) w_n(t) \frac{n^4 \pi^4}{L^4} \sin \left( \frac{n \pi x}{L} \right)
\]

\[
\int q(t) \sin \left( \frac{m \pi x}{L} \right) = \int -\mu(x) \frac{n^2 \pi^2}{L^2} w_n(t) \sin \left( \frac{n \pi t}{L} \right) \sin \left( \frac{m \pi x}{L} \right) dx + \int EI(x) w_n(t) \frac{n^4 \pi^4}{L^4} \sin \left( \frac{n \pi x}{L} \right) \sin \left( \frac{m \pi x}{L} \right) dx
\]

\[
Q_m = \frac{\int -\mu(x) \frac{n^2 \pi^2}{L^2} w_n(t) \sin \left( \frac{n \pi t}{L} \right) \sin \left( \frac{m \pi x}{L} \right) dx + \int EI(x) w_n(t) \frac{n^4 \pi^4}{L^4} \sin \left( \frac{n \pi x}{L} \right) \sin \left( \frac{m \pi x}{L} \right) dx}{\int \sin \left( \frac{m \pi x}{L} \right)}
\]

A graph of the resulting forcing for three modes in an undamaged scenario is seen in Figure 5-7.
Figure 5-7: Forcing of Undamaged OSV

After solving for both modal deflection and modal forcing, the additional substitutions were made to the governing equation

\[ A_{mn} = \int_{0}^{L} \sin\left(\frac{\pi mx}{L}\right) \sin\left(\frac{\pi nx}{L}\right) \mu(x) dx \]

\[ B_{mn} = c \int_{0}^{L} \sin\left(\frac{\pi mx}{L}\right) \sin\left(\frac{\pi nx}{L}\right) dx \]

\[ C_{mn} = \int_{0}^{L} EI(x) \sin\left(\frac{\pi mx}{L}\right) \left(\frac{\pi^4 n^4}{L^4}\right) \sin\left(\frac{\pi nx}{L}\right) dx \]

\[ Q_m(t) = \int Q_m \sin\left(\frac{\pi mx}{L}\right) dx \]

Final substitution into the governing equation yielded

\[ A_{mn} f''_n(t) + B_{mn} f'_n(t) + C_{mn} f_n(t) = Q_n(t) \]
Within the scenario-driven identification scheme, this equation is modified to

\[ A_{mn}f''_n(t) + B_{mn}f'_n(t) + (\tilde{C}_o + y_1\tilde{C}_1)f_n(t) = Q_n(t) \]

where \( f_n(t) \) is equal to the modal displacement that was solved for in Figure 5-6.

### 5.2.1 Higher Modes Resolution: Rescaling

On a first pass of the above formulation, the parameter selection remained insufficient for convergence. Rescaling was necessary to allow for convergence of the Ensemble Kalman Filter; this was done according to the following principle:

\[ \tau = \alpha t \]

\[ \alpha^2 x''_1 + \omega^2_{0,1} x_1 = 0 \]

\[ x''_1 + x_1 = 0 \]

\[ x''_2 + \frac{\omega^2_{0,2}}{\omega^2_{0,1}} x_2 = 0 \]

As a result, the governing ODE was defined as

\[ f''_n(t) + \frac{B_{mn}}{A_{mn}}f'_n(t) + \frac{C_{mn}}{A_{mn}}f_n(t) = \frac{Q_n(t)}{A_{mn}} \]

Due to the lack of damping within the model, the second term was dropped resulting in

\[ f''_n(t) + \frac{C_{mn}}{A_{mn}}f_n(t) = \frac{Q_n(t)}{A_{mn}} \]

\[ f''_n(t) + \frac{\pi^4 n^4 E(x)I(x)}{L^4 \mu(x)} f_n(t) = \frac{Q_n(t)}{A_{mn}} \]

The variable of interest is the variable modifying the \( f_n(t) \) which defines the fundamental frequency of the system. Setting

\[ \omega^2_{0,1} = \alpha = \sqrt{\frac{\pi^4 E(x)I(x)}{L^4 \mu(x)}} \]
the following equations were modified

\[
Q_n(t) = q_n(t)/\alpha^2
\]

\[
A_{mn}f''_n(t) + B_{mn}f'_n(t) + \left(\frac{\tilde{C}_o}{\alpha^2} + y_1\frac{\tilde{C}_1}{\alpha^2}\right)f_n(t) = Q_n(t)
\]

In trying to implement the above rescaling, the Ensemble Kalman Filter was still not able to converge on a solution. Through trial and error, it was determined that the appropriate factor to rescale the time domain by was

\[
\tau = \frac{1}{2500} t
\]

This rescaling was attempted by adding the scaling factor \(\frac{1}{2500}\) as an additional multiplicative factor to the rescaling shown above. However, this process as well was insufficient for convergence. The time rescaling to produce the results seen in Section 5.4 was accomplished by setting

\[
A_{mn}f''_n(\tau) + B_{mn}f'_n(\tau) + \left(\frac{\tilde{C}_o}{\alpha^2} + y_1\frac{\tilde{C}_1}{\alpha^2}\right)f_n(\tau) = Q_n(\tau)
\]

5.2.2 Kalman Filter Application and Noise Parameter Optimization

The Ensemble Kalman Filter was designed to determine displacement, velocity, and the damage factor measured in percent reduction. The set-up of the Ensemble Kalman Filter began with the identification of the desired state, \(F\), to include displacement \(f_n(t)\), velocity \(f'_n(t)\), and damage \(y_n\). The state was defined as

\[
F = [f_1 \quad f_2 \quad \ldots \quad f_n \quad f'_1 \quad f'_2 \quad \ldots \quad f'_n \quad y_1]^T
\]

Other initial definitions included the state covariance matrix \(N_S\), the observability matrix \(H\) where position was the only input, and the desired number of ensemble
members which was set to $N = 8000$. For $N_d = 2$, $N_S$ and $H$ were defined as

$$
N_S = \begin{bmatrix}
N_{EQS_1}^2 & 0 & 0 & 0 & 0 \\
0 & N_{EQS_2}^2 & 0 & 0 & 0 \\
0 & 0 & N_{EQS_1}^2 & 0 & 0 \\
0 & 0 & 0 & N_{EQS_2}^2 & 0 \\
0 & 0 & 0 & 0 & N_{PAR}^2
\end{bmatrix}
$$

(5.1)

$$
H = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0
\end{bmatrix}
$$

Within the Ensemble Kalman Filter, the true state is approximated by using the ensemble members. The prior ensemble is defined as a $[m \times N]$ matrix where $m$ is the number of states ($2N_d + 1$ in this case) and $N$ is the number of ensemble members (8000). For the length of an observation vector ($\tilde{z}$), defined to be the length of the deflection term, the posterior ensemble and state are updated in the following manner.

1. The prior ensemble is modified to include a time-influenced noise factor ($N_X$), comprised of a mean vector of zeroes and the state covariance matrix defined in Equation 5.1 consisting of parameter noise, $N_{PAR}$, and plant dynamics noise, $N_{EQS}$.

$$
F = F^{u}_1 + \sqrt{dt} N_X
$$

where $F^{u}_m$ is composed of the updated ensemble members as discussed in the Ensemble Member Update section.

2. Next, the posterior ensemble is calculated including a measurement data vector ($D$) that has had observation noise ($N_{OBS}$) added.

$$
F_{(k|k)} = F_{(k|k-1)} + [P_{(k|k-1)}H^T(HP^T_{(k|k-1)}H^T + N_{OBS}^2)^{-1}] [D - HF_{(k|k-1)}]
$$
3. The current state, \( F_{k|k} \), is defined as the mean of the ensemble members, \( \text{mean}(\mathbf{F}) \) and the current covariance, \( P_{k|k} \) is defined as the covariance of the ensemble members \( \text{cov}(\mathbf{F}) \).

**Ensemble Member Update**

Within the Ensemble Kalman Filter, a series of functions were set up to account for updates to the displacement and velocity ensemble members. The original ensemble state was modified using the coefficients from the ODE, \( A_n, B_n, \) and \( Q_n(t) \). \( C_o \) is the undamaged stiffness matrix and \( C_{mn} \) is the damaged stiffness matrix.

\[
\begin{bmatrix}
    f_n \\
    f'_n \\
    y_1 \\
\end{bmatrix}
= \begin{bmatrix}
    F^u_1 \\
    F^u_2 \\
    F^u_3 \\
\end{bmatrix}
= \begin{bmatrix}
    F_1 + dtF_2 \\
    F_2 + \frac{dt}{A_n}(-B_nF_2 - (C_o + F_3C_{mn})F_1 + Q_n(t)) \\
    F_3 \\
\end{bmatrix}
\]

As an example of the above code, for \( N_d = 2 \) modes, the implementation where \( \tilde{C} \) is the undamaged matrix and \( F_n \) is the ensemble representation is

\[
\begin{bmatrix}
    f_1 \\
    f_2 \\
    f'_1 \\
    f'_2 \\
    y_1 \\
\end{bmatrix}
= \begin{bmatrix}
    F_1 + dtF_3 \\
    F_2 + dtF_4 \\
    F_3 + dt(-\tilde{C}_{11}F_1 - F_5C_{11}F_1 - F_5C_{12}F_2 + Q_1(t)) \\
    F_4 + dt(-\tilde{C}_{22}F_2 - F_5C_{21}F_1 - F_5C_{22}F_2 + Q_2(t)) \\
    F_5 \\
\end{bmatrix}
\]
Noise Parameter Optimization

As previously discussed, noise parameter choices were critical for the proper evaluation of the SDI-EnKF model. The MAESTRO time-domain analysis returned sample data at $dt = 0.25$ seconds. The noise parameters were chosen to match with this time step. The results shown in Section 5.4 were generated with the choice of noise parameters shown in Table 5.2.

Table 5.2: Noise Parameters

<table>
<thead>
<tr>
<th>Mode</th>
<th>$N_{EQS}$</th>
<th>$N_{PAR}$</th>
<th>$N_{OBS}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mode 1</td>
<td>$5 \times 10^{-4}$</td>
<td>$5 \times 10^{-3}$</td>
<td>$5 \times 10^{-5}$</td>
</tr>
<tr>
<td>Mode 2</td>
<td>$5 \times 10^{-3}$</td>
<td>$3 \times 10^{-5}$</td>
<td></td>
</tr>
<tr>
<td>Mode 3</td>
<td>$2 \times 10^{-5}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

5.3 Damage Implementation

In order to simulate areas of damage within MAESTRO, internal stiffeners and bulkheads were removed from specified areas of the vessel. The MAESTRO structural analysis and time-domain analysis were re-run for the vessel for each damaged scenario. Upon damaging the model and the structural analysis, the hull longitudinal properties were reassessed. In a real-life analogue for structural damage, the mass density would not change significantly; the contributions of corrosion, wastage, and fatigue adding minimal changes to this material parameter. As such, it was assumed that the undamaged mass density of the vessel could be applied in all damaged cases. A table of damaged properties is shown in Table 5.3.

A pictorial implementation of this damage may be seen in Figure 5-8, which shows the changes made to the stiffness material property, $EI(x)$, as a function of location, $x$, for the entire hull.
Table 5.3: Damage Implementation

<table>
<thead>
<tr>
<th>Station</th>
<th>Location</th>
<th>Damaged Bow (% Change)</th>
<th>Damaged Midbody (% Change)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-1.6256 m</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0.889 m</td>
<td>-11.17</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>3.403 m</td>
<td>-3.04</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>5.9182 m</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>8.4328 m</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>10.9474 m</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>13.4620 m</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>8</td>
<td>15.9766 m</td>
<td>0</td>
<td>-1.80</td>
</tr>
<tr>
<td>9</td>
<td>18.4912 m</td>
<td>0</td>
<td>-1.86</td>
</tr>
<tr>
<td>10</td>
<td>21.0058 m</td>
<td>0</td>
<td>-1.85</td>
</tr>
<tr>
<td>11</td>
<td>23.5204 m</td>
<td>0</td>
<td>-1.85</td>
</tr>
<tr>
<td>12</td>
<td>26.0350 m</td>
<td>0</td>
<td>-1.85</td>
</tr>
<tr>
<td>13</td>
<td>28.5496 m</td>
<td>0</td>
<td>-1.84</td>
</tr>
<tr>
<td>14</td>
<td>31.0642 m</td>
<td>0</td>
<td>-1.62</td>
</tr>
<tr>
<td>15</td>
<td>33.5788 m</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>16</td>
<td>36.0934 m</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>17</td>
<td>38.6080 m</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>18</td>
<td>41.1226 m</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>19</td>
<td>43.6372 m</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>20</td>
<td>46.1518 m</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>21</td>
<td>48.6664 m</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
Figure 5-8: Damage Implementation to Stiffness Material Property
5.4 Validation Results

5.4.1 Undamaged Scenario

An initial calibration run was conducted for the undamaged hull. Results for deflection and forcing are shown in Figures 5-6 and 5-7. The SDI-EnKF’s estimated modal deflection and damage intensity are presented in Figures 5-9 and 5-10. The results of the undamaged scenario show that the damage identification for the validation model is within the model’s error ($\frac{1}{\sqrt{N}} = 1.12\%$), however when the initial guess is changed from $y_1 = 0$ to $y_1 = 0.1$ (Figure 5-11), the model fails to correct to the true value of $y_1 = 0$. Rather it stays at the initial guess with some variation towards the end of the time-domain analysis.

![Kalman Filter Position](image)

Figure 5-9: Undamaged Scenario Modal Position
Figure 5-10: Undamaged Scenario Damage Intensity, $y_1 = 0$

Figure 5-11: Undamaged Scenario Damage Intensity, $y_1 = 0.1$
5.4.2 Damaged Bow

The first damage analysis was conducted for a single area of damage implemented between Stations 2 through 4 at the bow. The modal response and modal forcing (Figures 5-12 and 5-13) were calculated as part of the model’s initial formulation. Notably, the modal forcing does not match the undamaged forcing (Figure 5-7). This does not match expected results, as the damage implementation should affect only the deflection; the wave forcing should remain the same for both the undamaged and damaged scenario. It appears that the forcing solution captures nonlinear effects present from the deflection solution. In order to validate the damaged model under similar sea actions to the undamaged model, the model was modified to incorporate the undamaged forcing. After sufficient rescaling, the SDI-EnKF was used to solve for deflection and intensity of damage. Results are shown in Figures 5-14 and 5-15. The code used to generate these results is included in Appendix B.

Figure 5-12: Modal Deflection with Damaged Bow
Figure 5-13: Modal Forcing with Damaged Bow

Figure 5-14: Position Response for \( n = 3 \), Measuring \( N = 3 \) Modes
Figure 5-15: Damage Response for n = 3, Measuring N = 3 Modes

The initial results suggest that given a long-enough time scale, the maximum extent of damage can be realized. However, towards the end of the time-domain simulation, the model begins fluctuating within a 5% band centered around the area of maximum damage. This fluctuation requires a longer time-domain analysis to determine the extent of this behavior; however, software limitations prevent a much longer analysis from being conducted. An additional contribution that may be complicating the damage verification is that the assumption of minimal change in mass density may not be valid with the removal of bulkheads to create change to the hull’s longitudinal properties.
5.4.3 Damaged Midbody

A second damaged run was conducted for a single area of damage between Stations 8 through 14 at the midbody. Similar to the damaged bow case, the modal forcing (Figure 5-17) does not show agreement with the undamaged case and the model was modified to include the undamaged modal forcing to have agreement between sea loading conditions. After sufficient rescaling, the SDI-EnKF was used to solve for deflection and intensity of damage. Results are shown in Figures 5-18 and 5-19.

![Modal Response](image)

Figure 5-16: Modal Deflection with Damaged Midbody
Figure 5-17: Modal Forcing with Damaged Midbody

Figure 5-18: Position Response for \( n = 3, \) Measuring \( N = 3 \) Modes
Similar to the damaged bow, the results of the damaged midbody show that given a long-enough time scale the maximum extent of damage can be realized. However, one problem complicating the damaged midbody damage identification is that the maximum extent of error (1.86%) is barely outside the model error (1.12%). This presents a resolution issue and the model’s performance here suggests that the model requires a higher degree of damage to detect the profile. Additionally, the fluctuation band towards the end of the time-domain requires a longer time-domain to analyze the extent of this behavior.

5.5 Discussion of Results

The results shown in Section 5.4 are promising. The behavior of the model will require analysis against a longer time-domain to evaluate the extent of fluctuations seen in Figures 5-15 and 5-19. It could be that hull girder vibrations are being captured in the modal deflection (such as in Figure 5-6) towards the end of the analyzed time-domain. This contribution then manifests as fluctuating changes in the latter
portion of the time-domain for damage identification. Additionally, the assumption of constant mass density was not further evaluated for validity. Contributions from the damage implementation process to changes in mass density could prevent the model from recognizing the maximum extent of damage more quickly. Despite these shortcomings, the validation results suggest that the maximum extent of damage can be realized. Modal deflection is captured within the model’s error and this information can be used to directly update degradation models for structural materials.

The results presented within this scenario-driven identification scheme take advantage of understood principles from naval architecture and nonlinear Kalman Filter methodology to identify areas of localized damage within a structure. The resolution that is achievable through this theoretical model suggest that the Ensemble Kalman Filter is an appropriate probabilistic method for damage identification within a continuous structural health monitoring set-up. While validation results require further evaluation, the end state suggests that this model can be of assistance in ship design and maintenance.
Chapter 6

Conclusion and Recommendations

6.1 Conclusions

This research focused on a theoretical model that could take advantage of the proliferation of sensors to identify localized damage without the need for visual inspection. This thesis did not focus on hardware limitations, financial implications, or shipboard integration of an evolved sensor suite. One could suggest that maritime industry practices would benefit more from a study of these topics than the further development of a statistical model. However, the development of a statistical model to discriminate specifically between an undamaged and damaged ship structure, specifically real-time localized assessment, is an area that has received little attention in technical journals. The resolution of identified areas of damage that is able to be achieved with this model is shown to be within 4\%, well below most classification societies’ required maintenance standards. The availability of diverse damage identification processes such as this model offer the ship-owner and ship-maintainer additional capabilities to make better informed decisions. For the US Navy, the consideration of future autonomous vessel designs as well as minimal crewing of her ships encourage the development of complementary maintenance procedures in which this probabilistic model is well-suited.
When coupled with a long-term structural health monitoring scheme, this model could assist with future ship design. Within the US Navy, current ship designs have focused on advancements in hull performance by changing to lightweight materials, such as aluminum, and non-conventional hullforms such as the Independence Class Littoral Combat Ship’s hybrid catamaran design. The design for these ships, adapted from high-speed ferries with average lifespans of ten years, will try to be extended to fit within the force composition for the next twenty years. In order to understand the impact to structural longevity, updated field methods will need to be employed to prevent the removal of in-service assets due to structural damage. Structural health monitoring offers a solution, not only for expanded maintenance procedures, but also for providing data that can update structural degradation models. The informed Ensemble Kalman Filter model represents a probabilistic method to update these models. Updated models providing realistic insights into today’s fleet will give future ship designers the trade-space to make appropriate decisions about the future fleet.

6.2 Suggestions for Future Work

Three major areas for future work to extend this thesis include the comparison to real sea-trials data, the reduction of uncertainty, and external data input.

Real sea-trials data were used in this thesis to validate the assumption of damaged areas along a hull. However, this data was not used as a direct input to the model. The next step in validation would include an analysis of this model’s performance to data collected from real sea-trials and an evaluation of the model’s limitations with such data.

As discussed in Chapter 4, the limitation to a scenario-driven identification scheme are the scenarios that are not considered. In the event that there is insufficient sensor coverage to capture data in areas of known risk, the model’s resolution will not only require re-scaling, but will require further validation. The reduction of uncertainty will be a product of the availability of data. This was not an area that
was approached in this thesis. However, given financial constraints or the risk of marine power distribution casualties, electronic sensors may not be able to provide huge quantities of data. The question of what is enough data to identify localized damage remains an outstanding question.

A final area of further research deals with the assumed inputs for the Ensemble Kalman Filter. The Ensemble Kalman Filter algorithm proposed in this research is a model that is based on material and geometric properties of the structure in question and an assumed starting point based on the understanding of ship loading according to naval architecture principles. There is currently no mechanism that allows for real-time input of damage identification from an external source. Regardless of the size of sensor suite installed onboard a ship, allowance for input from shipboard personnel’s visual inspection or maintenance would be prudent. This could be done by expanding the input ranges for damage identification, but risks the global resolution issues presented in Chapter 3.
Appendix A

Multiple Damage Areas

A.1 ODE Solver Function

```matlab
function fp=F(t,f,A,C,Q_diff,ww)
N=size(A,1);
q_d=Q_diff*sin(ww*t)';
fp=zeros(2*N,1);
fp(1:N)=f(N+1:2*N);
fp(N+1:2*N)=A^-1*(-C*f(1:N)+q_d);
```

A.2 Ensemble Kalman Filter

```matlab
function [xc,P] = SD2(t,zz,Q_diff,ww,Neqs,Nobs,Npar,x0,P0,C,...
Cund,A_diff,Nd,Md)
N=2000; % Ensemble Number
Dt=t(2)-t(1);
% Noise of equations
Ns = zeros(2*Nd+Md,2*Nd+Md);
```
Ns(1:Nd,1:Nd) = diag(Neqs.^2);
Ns(Nd+1:2*Nd,Nd+1:2*Nd) = diag(Neqs.^2);
Ns(2*Nd+1:2*Nd+Md,2*Nd+1:2*Nd+Md) = diag(Npar.^2);

P(:,:,1) = P0; % Initial Covariance
xc(:,1)= x0(:,1)'; % Initial State Estimate
H=[eye(Nd),zeros(Nd,Nd),zeros(Nd,Md)];

XX = mvnrnd(x0,P0,N)'; %Initial Ensemble

for k=2:length(t); %Each time step
    xt=xc(:,k-1); % Current a posteriori state [11xtime]
    Pt=P(:,:,k-1);

    NX = mvnrnd(zeros(1,length(x0)),Ns,N)';
    %Prior Ensemble Update as an SDE (White noise addition)
    XX = FF(Q_diff,ww,t(k),XX,Dt,Nd,C,Cund,A_diff,N,Md)+... 
         sqrt(Dt)*NX;

    %Data vector based on observation + noise
    D = mvnrnd(zz(k,:),diag(Nobs).^2,N)'; %[5x200]

    %Posterior Ensemble
    XX = XX+Pt*H'*(H*Pt*H'+diag(Nobs.^2))^-1*(D-H*XX);
    xc(:,k) = mean(XX'); %State update
    P(:,:,k) = cov(XX'); %Covariance update
end
end

%State Update
function y = FF(Q_diff,ww,ti,x,Dt,Nd,C,Cund,A,N,Md)
    VForc=0*Q_diff(1:Nd,:)*sin(ww*ti)';
    MM=zeros(Nd,N);
    for k=1:Md
        MM=MM+(C(:,:,k)*x(1:Nd,:)).*repmat(x(2*Nd+k,:),Nd,1);
    end
    y = MM*(1:Nd,:);
43  end
44
45  y(1:Nd,:) = x(1:Nd,:) + Dt * x(1+Nd:2*Nd,:);
46  y(Nd+1:2*Nd,:) = x(1+Nd:2*Nd,:) + Dt * A^-1 * (-Cund * x(1:Nd,:) - MM... + repmat(VForc,1,N));
47  y(2*Nd+1:2*Nd+Md,:) = x(2*Nd+1:2*Nd+Md,:);
48  end

A.3 Damage Detection with Ensemble Kalman Filter

```
1  \% Ensemble Kalman Filter
2  clear all; close all;
3
4  \% Givens
5  \% Beam Properties
6  L = 92; \% [m] Length Water Line HSV-2 (92); Frame Spacing 3.94mx5
7  Beam = 4.5; \% [m] Hull Beam HSV-2 (Full Beam is 26.6 m)
8  D = 3.43; \% [m] Draft HSV-2
9
10  \% Material Properties
11  I_n = (1/12) * Beam * D^3; \% [m^4] Second Moment of Area
12  E = 7.1 * 10^10; \% [N/m^2] Elastic Modulus of 6082-Aluminum
13  rho = 2710; \% [kg/m^3] Density of 6082-Aluminum
14  mu = Beam * D * rho; \% [kg/m] Mass/Unit Length of Aluminum
15
16  \% Rescaling Factor
17  EIm = E * I_n / mu;
18  aa = sqrt(EIm / L^4 * pi^4);
19
20  \% Spatial Discretization
21  sampling = 300; \% Sampling Interval along the beam
```
dx = L/(sampling-1); % Discretization of beam
xspan = 0:dx:L;
dn = length(xspan);

% Input Damage Area - Multiple areas
Md = 3; % number of potential damage locations
% Definition of 1st damage location/profile
theta1 = zeros(1,dn); %[m^4/s^2]
for kk = round(0.6*dn):1:round(0.8*dn)
    theta1(kk) = -1+theta1(1,1);
end
EI_x(1,:) = theta1;

% Definition of 2nd damage location/profile
theta1 = zeros(1,dn); %[m^4/s^2]
for kk = round(0.1*dn):1:round(0.2*dn)
    theta1(kk) = -1+theta1(1,1);
end
EI_x(2,:) = theta1;

% Definition of 3rd damage location/profile
theta1 = zeros(1,dn); %[m^4/s^2]
for kk = round(0.3*dn):1:round(0.45*dn)
    theta1(kk) = -1+theta1(1,1);
end
EI_x(3,:) = theta1;

% Measurements
% Setup
N = 2; %Number of Desired Modes
Time = 200;
Dt = 0.01;
tspan = 0:Dt:Time;

%Modal Forcing Term Q_n(t)= q(x,t)*g_n(x)
x0=xspan;
ww=[1,5]/aa; %Excitation frequencies

% Spatial content for freq ww(1)/freq ww(2)
fun_q(:,1)=(1.0*sin(pi*x0/L)+0.9*sin(3*pi*x0/L)+0.8*sin(5*pi*x0/L)+1.0*sin(9*pi*x0/L))/aa^2;

fun_q(:,2)=(1.0*sin(2*pi*x0/L)+0.9*sin(4*pi*x0/L)+1.0*sin(6*pi*x0/L)+0.7*sin(8*pi*x0/L)+1.0*sin(10*pi*x0/L))/aa^2;

% Solve for Modal Response
for n = 1:N
    for m = 1:N; %Non-Homogenous Case
        A_diff(m,n) = sum(sin(pi*m*x0/L).*sin(pi*n*x0/L))*dx;
        C_und(m,n) = EIm/aa^2*sum(sin(pi*m*x0/L).*(n^4*pi^4)/L^4.*sin(pi*n*x0/L))*dx;
        %Damaged Case
        for j=1:Md
            C_diff(m,n,j) = EIm/aa^2*sum(sin(pi*m*x0/L).*(n^4*pi^4)/L^4.*sin(pi*n*x0/L).*EI_x(j,:))*dx;
        end
    end
    for i=1:length(ww)
        Q_diff(n,i)=sum(fun_q(:,i).*sin(n*pi*x0'/L))*dx;
    end
end

% ODE Solution
% A*f''+C*f = Q
C_dam = C_und + .3*C_diff(:,;2)+ .2*C_diff(:,;3);
for i = 1:1:N %For Desired Modes
    i
    %DOUBLE-CHECK INITIAL CONDITIONS
    options = odeset('RelTol',1e-12,'AbsTol',1e-12*ones(1,2*N));
    [t,f]=ode45(@(t,f) myODE(t,f,A_diff,C_dam,Q_diff,ww),
    tspan,zeros(2*N,1),options);
% Store Results
Results = f(:,1:N);  % Actual Measurements for Displacement
plot(t,Results(:,i)); hold on; grid on;
% break
end
legend('Mode 1','Mode 2',' Mode 3', 'Mode 4', ' Mode 5');
print -depsc ModalCoefficients

%% Filter
figure
for ii = 1:1:N
    subplot(2,1,1);
    plot(t,Results(:,ii));
    hold on; grid on;
    title('Real Results');

    subplot(2,1,2);
    [envHigh, envLow] = envelope(Results(:,ii),16,'peak');
    envMean(:,ii) = (envHigh+envLow)/2;
    plot(t,envMean(:,ii)); hold on; grid on;
    title('Filtered Results [Mean]'); xlabel('Time (s)');
end

%% Ensemble Kalman Filter [f,f',y]
Nd = N;
zz = envMean(:,1:Nd);  % Observation Vector within the Damaged Area

% Initial Guesses: Parameter Selection
Npar(1:Md) = 5*1*10^-3;  % Parameter noise
for k = 1
    Neqs(k) = 1*10^-4;  % System noise
    Nobs(k) = 1*5*10^-3;  % Observation noise
end
for k = 2
    Neqs(k) = 1*10^-4;  % System noise
Nobs(k) = 1*10^-3; %Observation noise
end
for k = 3
Neqs(k) = 1*10^-4; %System noise
Nobs(k) = 1*10^-3; %Observation noise
end

% Main Loop EnKF
% State guess
State = zeros(2*Nd+Md,1);
%Covariance Guess
P0 = eye(2*Nd+Md)*10^-2;

[xc_E,P]=SD2(t,zz(:,1:Nd),Q_diff,ww,Neqs(1:Nd),...
Nobs(1:Nd),Npar,State,P0, C_diff(1:Nd,1:Nd,1:Md),...
C_und(1:Nd,1:Nd),A_diff(1:Nd,1:Nd),Nd,Md);

% Data-Gathering & Plotting
for k = 1:Nd
Position(:,k) = xc_E(1+(k-1),:)

% Plotting
subplot(3,1,k); plot(t,Results(:,k)); hold on; ...
plot(t,Position(:,k));
title('Position'); grid on;
xlabel('Time'); axis 'auto y';
legend('Exact','Ensemble KF');

end

Reduction = xc_E(2*Nd+1:2*Nd+Md,:);
figure;
hold on; plot(t,Reduction);
title('Damage'); grid on;
xlabel('Time'); ylabel('y_1'); axis 'auto y';
legend('Exact','Ensemble KF');
k = 1:size(Reduction',1);

legend('y_1: 0.6L-0.8L', 'y_2: 0.1L-0.2L', 'y_3: 0.3L-0.45L');

print -depsc Damage
Appendix B

Validation Code

B.1 Ensemble Kalman Filter

```matlab
function [xc,P] = SD1(t,zz,Q_diff,Neqs,Nobs,Npar,x0,P0,C,...
Cund,A_diff,Nd)

N=8000;  %Ensemble Number
Dt=t(2)-t(1);

% Noise of equations
Ns = zeros(2*Nd+1,2*Nd+1);
Ns(1:Nd,1:Nd) = diag(Neqs.^2);
Ns(Nd+1:2*Nd,Nd+1:2*Nd) = diag(Neqs.^2);
Ns(2*Nd+1,2*Nd+1) = Npar(1).^2;

P(:,:,1) = P0;  % Initial Covariance
xc(:,1)= x0(:,1)';  % Initial State Estimate: [3*Ndx1] [f1 f1' y]
H=[eye(Nd),zeros(Nd),zeros(Nd,1)];

XX = mvnrnd(x0,P0,N)';  %Initial Ensemble

for k=2:length(t);  %Each time step

%k
xt=xc(:,k-1);  % Current a posteriori state [11xtime]
```
\texttt{Pt=P(:,:,k-1);} 
\texttt{NX = mvnrnd(zeros(1,length(x0)),Ns,N)';} 
\texttt{\% Add white noise} 
\texttt{XX = FF(Q\_diff,XX,Dt,Nd,C,Cund,A\_diff,N,k)+sqrt(Dt)*NX;} 
\texttt{\% Data vector based on observation + noise} 
\texttt{D = mvnrnd(zz(k,:),diag(Nobs).^2,N)';} 
\texttt{\% Posterior Ensemble} 
\texttt{JJ = H*Pt*H'+diag(Nobs.^2);} 
\texttt{XX = XX+Pt*H'*inv(JJ)*(D-H*XX);} 
\texttt{xc(:,k) = mean(XX'); \% State update} 
\texttt{P(:,:,k) = cov(XX'); \% Covariance update} 
\texttt{end} 
\texttt{end} 
\texttt{\% State Update} 
\texttt{function y = FF(Q\_diff,x,Dt,Nd,C,Cund,A,N,k)} 
\texttt{VForc=Q\_diff(1:Nd,k);} 
\texttt{MM=(C*x(1:Nd,:)).*repmat(x(2*Nd+1,:),Nd,1); \% Damage matrix} 
\texttt{y(1:Nd,:)=x(1:Nd,:)+Dt*x(1+Nd:2*Nd,:);} 
\texttt{y(Nd+1:2*Nd,:)=x(1+Nd:2*Nd,:)+Dt*A^-1*(-Cund*x(1:Nd,:)-...} 
\texttt{-MM+repmat(VForc,1,N));} 
\texttt{y(2*Nd+1,:) = x(2*Nd+1,:);} 
\texttt{end} 

\section*{B.2 Damage Detection with Ensemble Kalman Filter}
clear all; close all;

%% Givens
%
%% Unit Conversion
tonne2Newton = 9806.65;

%% Beam Properties
L = 50.292; % [m] Length Water Line OSV
Beam = 11.03; % [m] Beam OSV
D = 1.49; % [m] DWL

%% Material Properties
I_n = (1/12)*Beam*D^3; % [m^4] Second Moment of Area
E = 2.04*10^11; % [N/m^2] Elastic Modulus of ST-24
rho = 7850; % [kg/m^3] Density of ST-24

%% Time
N = 3; % Number of Desired Modes
Time = 850;
Dt = 0.25;
tspan = 0: Dt: Time;
T = length(tspan);

Time2 = 850;
Dt2 = 0.25;
tspan2 = 0: Dt2: Time2;
T2 = length(tspan2);

%% Properties per Location: Undamaged Case
% Speed=0.00 knots; Heading=180.00 Degrees, Wave-Length=156(m),
% Wave-Period=10.000(s)

%% Moments and Shear Force
S0 = xlsread('Undamaged_850.xlsx');
Location0 = S0(:,1); % m
Location0(isnan(Location0)) = [];
TimeData0 = S0(:,2); % s
TimeData0(isnan(TimeData0)) = [];  
ForceX0 = S0(:,3); % tonne  
VSF0 = S0(:,4); % Vertical Shear Force (tonne)  
VBM0 = S0(:,8); % Vertical Bending Moment (tonne-m)  
VBM0(isnan(VBM0)) = [];

% I(x) and mu(x)  
S0Iy = xlsread('Undamaged.xlsx','Iyy');  
xLoc = S0Iy(:,2); %m  
Iyy = S0Iy(:,5); %m^4  
S0mu = xlsread('Undamaged.xlsx','Mass');  
xLocmu = S0mu(:,1); % m  
mu = S0mu(:,2); % tonne

%Spatial Discretization  
Location_Sampling0 = unique(Location0,'rows','first');  
Location_Sampling0(isnan(Location_Sampling0)) = [];

dX0 = L/(sampling0-1); % Discretization of beam  
xspan0 = Location_Sampling0';  
discretize = sampling0;

Moment_Matrix0 = [TimeData0 Location0 VBM0]; %[sec m tonne-m]  

%Inertia and Mass Density: Process to Correct Units  
Moment_Matrix0(:,4) = zeros(length(TimeData0),1);  
I_Matrix0 = [xLoc Iyy]; %[m m^4]  
Inew0 = repmat(I_Matrix0,T2,1); Inew0 = sortrows(Inew0,1);  
Moment_Matrix0(:,4) = Inew0(:,2);

Moment_Matrix0(:,5) = zeros(length(TimeData0),1);  
mu00 = [xLocmu (1000*mu)./xspan0']; %[kg/m]  
munew0 = repmat(mu00,T2,1); munew0 = sortrows(munew0,1);  
Moment_Matrix0(:,5) = munew0(:,2);

% % Read Damaged Properties
S = xlsread('Damaged_Bow_850.xlsx');
Location = S(:,1); % m
Location(isnan(Location)) = [];
TimeData = S(:,2); % s
TimeData(isnan(TimeData)) = [];
ForceX = S(:,3); % tonne
VSF = S(:,4); % Vertical Shear Force (tonne)
VBM = S(:,8); % Vertical Bending Moment (tonne-m)
VBM(isnan(VBM)) = [];

% I(x) and mu(x)
S1Ix = xlsread('Bow_Properties.xlsx','Iyy');
S2Ix = xlsread('Midbody_Properties_2.xlsx','Iyy');

xLoc1 = S1Ix(:,2); xLoc2 = S2Ix(:,2); % m
Iyy1 = S1Ix(:,5); Iyy2 = S2Ix(:,5); % m^4

EI_1 = E*Iyy1; EI_2 = E*Iyy2;

plot(xLoc,E*Iyy); hold on; grid on;
plot(xLoc1, EI_1, '*'); plot(xLoc2,EI_2, '^');
axis([0 8.4328 0 9e+11]);
legend('Undamaged','Damaged Bow','Damaged Midbody',... 'Location','Best');
xlabel('Location (m)'); ylabel('EI(x)(N-m^2)');
print -depsc DamagedBowDescription

% Spatial Discretization
Location_Sampling = unique(Location,'rows','first');
Location_Sampling(isnan(Location_Sampling)) = [];
sampling = length(Location_Sampling);
dx = L/(sampling-1); % Discretization of beam
xspan = Location_Sampling';
discretize = sampling;

Moment_Matrix = [TimeData Location VBM]; %[sec m tonne-m]

% Inertia and Mass Density: Process to Correct Units
Moment_Matrix(:,4) = zeros(length(TimeData),1);
I_Matrix = [xLoc Iyy];
%
I_Matrix = [xLoc Iyy];
Inew = repmat(I_Matrix,T,1); Inew = sortrows(Inew,1);
Moment_Matrix(:,4) = Inew(:,2);
Moment_Matrix(:,5) = zeros(length(TimeData),1);
mu0 = [xLocmu (1000*mu)./xspan'];
%
mu0 = [xLocmu (1000*mu)./xspan']; %kg/m
munew = repmat(mu0,T,1); munew = sortrows(munew,1);
Moment_Matrix(:,5) = munew(:,2);
%
0. Solve for Deflection
VBMund_xt = reshape(VBM0,T2,21)'; %tonne-m; 1; Undamaged Data
VBM_xt = reshape(VBM,T,21)'; %tonne-m; 1; Bow Damaged Data
EI_xund = reshape(Moment_Matrix0(:,4),T2,21)'; %N-m^2; 10^11;
EI_x = reshape(E*Moment_Matrix(:,4),T,21)'; %N-m^2; 10^11;
for m1=1:N
    for m2=1:N
        MM_und(m2,m1)=dx*nansum(EI_xund(:,1).*...
            sin(m1*pi*xLoc/L).*sin(m2*pi*xLoc/L).*...
            m1^2*pi^2/L^2); %N-m
        MM(m2,m1)=dx*nansum(EI_1(:,1).*sin(m1*pi*xLoc/L).*...
            sin(m2*pi*xLoc/L).*m1^2*pi^2/L^2); %N-m
    end
end
for i=1:size(VBM_xt,2)
    for j=1:N
        tt(j)=sum(sin(j*pi*xLoc/L).*tonne2Newton.*...
            VBM_xt(:,i))*dx; %N-m^2
    end
    ttm=MM^-1*tt'; %m
    md(:,i)=ttm';
end
for i=1:size(VBMund_xt,2)
    for j=1:N
        tt_und(j)=sum(sin(j*pi*xLoc/L).*tonne2Newton.*...
            VBMund_xt(:,i))*dx; %N-m^2
end
ttu=MM^{-1} \cdot tt\_und';
md\_und(:,i)=ttu';
end

% Plot modal deflection
for i=1:N
    subplot(3,N,i); plot(tspan,md(i,:)); grid on;
    title(['Damaged Mode ', num2str(i)]);
    subplot(3,N,i+3); plot(tspan2,md\_und(i,:));
    title(['Undamaged Mode ', num2str(i)]); grid on;
    subplot(3,N,i+6); plot(tspan,md(i,:),tspan2,md\_und(i,:));
    axis([0 850 -10^{-5} 10^{-5}]); axis 'auto y';
    title(['Combined Mode ', num2str(i)]); grid on;
end
suptitle('Modal Deflection');
print -depsc Deflection_Bow

% 1. Solve for q\_m(t)
miou\_x = reshape(Moment\_Matrix(:,5),T,21)'; \%kg/m; 10^2
miou\_x0 = reshape(Moment\_Matrix0(:,5),T2,21)'; \%kg/m; 10^2
%Assumption here is that despite the change in mass to
%create a validation data set, the mass will not
%significantly decrease

%Spatial Dependencies
%Damaged deflection goes with damaged forcing- Change the EI in the
%spatial term for different damaged scenarios
for m3 = 1:N
    for m4 = 1:N
        mass(m3,m4,:) = ((-miou\_x(:,1)).\*m3^2\*pi^2/L^2).\*sin(m4\*pi\*xLoc/L);
        %[NxNx21]
        time\_factor(m3,:) = sin(m3\*pi\*tspan/L); \%[Nx241]
        w4(m3,m4,:) = EI\_1(:,1).\*sin(m3\*pi\*xLoc/L).\*sin(m4\*pi\*xLoc/L).\*m3^4\*pi^2/L^4;
        % [NxNx21]
        q\_const(m4) = dx*sum(sin(m4\*pi\*xLoc/L)); \%[Nx1]
for l = 1:size(miou_x,2)
    for z = 1:N
        MASS = reshape(mass(z,:,:),21,N);
        W4 = reshape(w4(z,:,:),21,N);
        qm_t(z,l) = dx*sum(MASS*md(:,l).*
            sin(z*pi*time_factor(z,l)/L))+
            +dx*sum(W4*md(:,l))*(q_const(z)^-1)';
    end
end

for m3 = 1:N
    for m4 = 1:N
        mass0(m3,m4,:) = ((-miou_x0(:,1)).*m3^2*pi^2/L^2).*
            sin(m4*pi*xLoc/L); %[NxN21]
        time_factor0(m3,:) = sin(m3*pi*tspan2/L); %[N241]
        w40(m3,m4,:) = EI_xund(:,1).*sin(m3*pi*xLoc/L).*
            sin(m4*pi*xLoc/L)...  
            .*m3^4*pi^4/L^4; % [NxN21]
        q_const0(m4) = dx*sum(sin(m4*pi*xLoc/L)); %[Nx1]
    end
end

for l = 1:size(miou_x0,2)
    for z = 1:N
        MASS0 = reshape(mass0(z,:,:),21,N);
        W40 = reshape(w40(z,:,:),21,N);
        qm_t0(z,l) = dx*sum(MASS0*md_und(:,l).*
            sin(z*pi*time_factor0(z,l)/L))+
            +dx*sum(W40*md_und(:,l))*(q_const0(z)^-1)';
    end
end

% Plot modal forcing
figure
for i=1:N
    subplot(3,1,i); plot(tspan,qm_t(i,:)); grid on;
    title(['Mode ', num2str(i)]);
    axis([0 850 0 1000]); axis 'auto y';
    ax1 = subplot(N,1,N); xlabel(ax1,'Time (sec)');
    ylabel(ax1,'Forcing (N/m)');
end

supptitle('Modal Forcing');
print -depsc Forcing_Bow

%% 2. Solve for Modal Response
% Rescaling Factor- Undamaged
EIm = max(EI_1(:,1))./max(miou_x(:,2)); %m^4/s^2
aa = sqrt(pi^4*EIm/L^4); % s^-1
scaling = aa^2;
x0 = xspan(1:end);

DamageActualBow = ((EI_x(:,1)-EI_1(:,1))./EI_x(:,1));
Stiffness = -1*(EI_x(:,1)-EI_1(:,1)).*ones(21,1);
A = find(Stiffness);
Stiffness2 = zeros(21,1);
Stiffness2(A,1) = DamageActualBow(A,1);

%Non-Homogenous Case: Scaled
for n = 1:N
    for m = 1:N;
        A_diff(m,n) = dx*sum((mu0(:,2)'.*sin(pi*m*x0/L)').*...
            sin(pi*n*x0/L)); % kg
        C_und(m,n) = dx*nansum(EI_x(:,1).*(((sin(pi*m*x0/L).*...
            (n^4*pi^4/L^4))'*sin(pi*n*x0/L'))))./scaling; % m
    end
%Damaged Case
C_diff(m,n) = dx*nansum((Stiffness2.*EI_x(:,1)).*...
    (((sin(pi*m*x0/L).*...
        (n^4*pi^4/L^4))'*sin(pi*n*x0/L'))))./scaling; % m
end
for i=1:length(tspan)
    Q_diff(n,i)=dx.*sum(qm_t0(n,i).*sin(n*pi*x0'/L))/aa^2;
end

%% Plot Rescaled Modal Response
figure
for i=1:N
    subplot(N,1,i); plot(tspan,md(i,:)'./(scaling)); hold on; grid on;
    title(['Mode ', num2str(i)]); axis([0 850 -1 1]); axis 'auto y';
    %legend('Deflection','ODE Solver','Location','BestOutside');
    ax = gca; ax.YAxis.TickLabelFormat = '%,.2f';
    ax1 = subplot(N,1,N); xlabel(ax1,'Time (sec)');
    ylabel(ax1,'Response');
end
suptitle('Modal Response');
print -dep sc Deflection_Rescaled

%% 3. Set Up Ensemble Kalman Filter [f,f',y]
Nd = N;
Results = (md(1:Nd,:)'./(scaling));%Truth
% Observation Vector within the Damaged Area
zz = (md(1:Nd,:)./(scaling))';

%% Initial Guesses: Parameter Selection
Npar(1) = 1*5*10^-4; %Parameter noise
for k = 1
    Neqs(k) = 5*10^-3; %System noise
    Nobs(k) = 1*5*10^-5; %Observation noise
end
for k = 2
    Neqs(k) = 5*10^-3; %System noise
    Nobs(k) = 1*3*10^-5; %Observation noise
end
for k = 3:N
Neqs(k) = 5*10^-3; %System noise
Nobs(k) = 1*2*10^-5; %Observation noise
end

% Main Loop
%State Guess
for l=1:1:Nd
    State_Pos(l) = 0;
    State_Vel(l) = 0;
    Damage = 0.0; %Damage- Percent reduction (y)
    Guess(:,l) = [State_Pos(l)'; State_Vel(l)'];
end

Guess2 = [reshape(Guess,[],1)];
State = [Guess2; Damage]; %[f f' y]

% Covariance Guess
P0 = eye(2*Nd+1)*10^-12;

% EnKF
[xc_E,P] = SD1(t./2500,zz(:,1:Nd),Q_diff,Neqs(1:Nd),Nobs(1:Nd),...
    Npar(:,1),State,P0,C_diff(1:Nd,1:Nd),C_und(1:Nd,1:Nd),...
    A_diff(1:Nd,1:Nd),Nd);

% 4. Data-Gathering & Plotting EnKF
for k = 1:Nd
    Position(:,k) = xc_E(1+(k-1),:);
    subplot(N,1,k); plot(t,Results(:,k),'LineWidth',0.75);
    grid on; hold on; axis([0 tspan(end) -3 3]); axis 'auto y';
    plot(t,Position(:,k),'-.','LineWidth',0.75);
    grid on; hold on; axis([0 tspan(end) -3 3]); axis 'auto y';
    title(['Mode ', num2str(k)]);
end
ax = gca;
    ax.YAxis.TickLabelFormat = '%,.2f';
ax1 = subplot(N,1,N); xlabel(ax1,'Time (sec)');
ylabel(ax1,'Position');
ax2 = subplot(3,1,1); axis([0 tspan(end) -4 4]);
legend('Exact', 'Ensemble KF', 'location','northeast',...
    'orientation','horizontal');
suptitle('Kalman Filter Position')
print -depsc Position_850

DamageActualBow = ((EI_x(:,1)-EI_1(:,1))./EI_x(:,1));

Reduction = xc_E(2*Nd+1,:);
figure;
subplot(2,2,[1,2]); plot(t,Reduction(:));
xlabel('Time'); ylabel('y_1'); axis 'auto y';
%legend('EnKF Results','Location','NorthWest');
title('EnKF Results');
axis([0 tspan(end) 0 1000]); axis 'auto y'; grid on;
subplot(2,2,[3,4]); plot(xspan,DamageActualBow,'LineWidth',.75)
hold on; plot(xspan,zeros(21,1),'LineWidth',.75); grid on;
title('Damage Profile Over Space');
xlabel('Location (m)'); ylabel('EI_x (%Original)');
legend('Damaged (y_1 = 1)',...
    'Undamaged (y_1 = 0)','Location','NorthEast');
axis([0 xspan(end) 0 1000]); axis 'auto y'; grid on;
suptitle('Damaged Bow');
print -depsc Damage_850


